The Money-Demand with Random Output and Limited Access to Debt

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Abstract

The money-demand of the economy is characterised, when national output is random and investors cannot attract any level of debt at any moment without incurring in additional costs. The *optimal* cash-balance is then expressed as the probability-quantile (or Value-at-Risk) of the series of capital returns on income, and in this way, it is explicitly determined by risk. As a consequence, the interest-rate-elasticity depends on the kind of risks and expectations, in such a way that the more unstable the economy, the greater the interest-rate-elasticity of the money demand. Therefore, the effectiveness of monetary policy is increased by diminishing the variability of output. Moreover, since flows of capital can affect the riskiness of financial securities by modifying the amounts involved in transactions, part of the adjustment to reestablish the short-run monetary equilibrium may be performed through volatility shocks. Finally, for different parametrisations of risks, aggregated parameters are expressed as the weighted average of sectorial estimations, so that multiple equilibria of the economy are allowed.

Key words: Money demand; Monetary policy; Economic capital; Distorted risk principle; Value-at-Risk.

IME-Classification: IE10, IE20, IE50.
JEL-Classification: E41, E44, E52, G11.

1 Introduction

According to the Keynes' (1935) *liquidity-preference* proposition, the demand for cash-balances is positively affected by the level of income and negatively affected by the return offered by a class of *non-risky* instruments. The first part of the proposition is a consequence of the *transactions-motive*. To explain the effect of the rate of interest, Keynes emphasises the role played by the *speculative-motive*. Thus, decision-makers expecting interest-rates to rise demand fewer risk-free securities in order to avoid capital losses. By contrast, when interest-rates are expected to fall, more bonds are demanded — in this way, once interest-rates collapse and the prices of bonds rise, capital gains can be attained. Therefore, fewer provisions are maintained for higher levels of the rate of interest and vice versa. A functional expression can then be provided for the money-demand. Thus, if Y and y respectively represent the *nominal* and *real* incomes, in such a way that $Y = P \cdot y$, where P denotes the level of prices, the liquidity demand is given by:

$$L \;=\; P\, y \cdot l(r) \;=\; Y \cdot l(r) \quad with \quad \frac{dl(r)}{dr} < 0$$

The *liquidity-preference* function $l(r)$ expresses the ratio between demanded cash-balances and nominal income. It is not likely to be constant but it may change slowly over time. The inverse ratio of the liquidity preference function is called *velocity of money*.

Let us denote by M the aggregated supply of money in the following, which although mainly controlled by the central bank, can also be altered by private investors in open capital markets. It is traditionally related to a class of *narrow* money, denoted by $M1$, containing *currency held by the public*, though other monetary aggregates, such as $M2$, which includes saving and small-denomination time deposits, as well as retail mutual funds, and $M3$, which adds mutual funds, repurchase agreements and large-denomination time deposits, are also available. In recent times, the class of *Money Zero Maturity* instruments has been introduced, denoted by MZM. The short-run monetary equilibrium is then determined by forcing the demanded cash-balance to be equal to the total stock of money:

$$M \;=\; P\, y \cdot l(r) \;=\; Y \cdot l(r) \tag{1}$$

A change in the nominal quantity of money will require a change in one or more of the variables determining the liquidity demand, i.e. P, y or r, in order to reestablish the monetary equilibrium. When prices are rigid in response to short-run fluctuations — and there is no distinction between real and nominal magnitudes — the whole adjustment is performed in $l(r)$. If additionally liquidity preference is *absolute*, i.e. if investors are satisfied at a single level of the rate of interest, the amount of money might change without a change in either nominal income or interest-rates. Under such circumstances, monetary policy is useless for dealing with short-run fluctuations.

The situation is different if prices are flexible and liquidity preference is *non-absolute*. Thus, let us suppose, as Friedman (1970) does, that prices adjust more rapidly than quantities, so rapidly in fact that the price adjustment can be regarded as *instantaneous*. A monetary expansion then produces a new equilibrium involving a higher price for the same quantity, the higher this response the more inelastic the money demand. In the short-run, production is encouraged until prices are reestablished at their original level. In the long-run, new producers enter the market and existing plants are expanded. Throughout the process, *it takes time for output to adjust but no time for prices to do so.*

In any intermediate case, monetary authorities can *partially* stimulate the real side of the economy by modifying the stock of money — through the so-called *transmission-mechanism*. Thus, after a monetary expansion — performed by means of operations in (primary) capital markets — the level of prices will be pushed to increase, at the time that the rate of interest will decrease in order to satisfy the liquidity-preference of investors at the new stock of cash — the higher this effect the higher the interest-rate elasticity of the money-demand. A new equilibrium will then be attained, involving a higher real income only if the adjustments for inflation and liquidity-preference do not overcome the increment of the money-supply. This is the reason why many central banks prefer to adopt *inflation-targeting* or *price-stabilisation* policies. Under such circumstances, the efficacy of monetary policy purely depends on the stability and the interest-rate elasticity of the money-demand.

There is a consensus among researchers about the existence of a stable long-run relationship, though fluctuations of cash balances in the short-run remain unexplained. Episodes like the *missing money* in the mid-seventies (documented by Goldfeld, 1976), the great velocity decline in the early eighties, followed by the expansion of narrow money in the mid-eighties, or the *velocity puzzle* of the mid-nineties, still lack a satisfactory explanation (see Ball, 2001; Carpenter and Lange, 2002; Teles and Zhou, 2005). In accounting for such drawbacks, recent literature has focused on *uncertainty*, which is supposed to have been incremented after 1980 due to deregulation and financial innovations (Atta-Mensah, 2004; Baum et al., 2005; Carpenter and Lange, 2002; Choi and Oh, 2003; Greiber and Lemke, 2005). Calza and Sousa (2003) have instead centered their attention on idiosyncracy and aggregation. Deregulation and financial innovation are also given as arguments to support the role of the opportunity cost in accounting for unexplained fluctuations (Ball, 2002; Collins and Edwards, 1994; Duca, 2000; Dreger and Wolters, 2006; Teles and Zhou, 2005). According to this view, a stable long-run relationship exits and movements of the interest rate can explain all short-run episodes, as long as the right monetary aggregate is used. Teles and Zhou (2005) additionally argue that the maturity of financial securities determines liquidity as well, in such a way that the class MZM, which contains Zero-Coupon instruments, provides an appropriate reference for the opportunity cost of money.

An *extended* or *generalised* model is presented in this paper, according to which liquidity-preference is explicitly determined by *uncertainty* and *information*. First the cash demand of a single representative investor is obtained. The classic reference in this respect is Tobin (1958), who assuming *perfect* conditions in capital markets, such that, in particular, lending and borrowing are allowed without restriction, proves that, as long as decision-makers show aversion-to-risk, a single portfolio — containing risk and equity — maximises expected utility and so liquidity-preference is obtained as behaviour towards risk. In the extended model, the hypothesis of perfect competition is abandoned. Investors are thus supposed to face constraints on liquidity and accordingly, they cannot attract any amount of capital without incurring in additional costs — in other words, they have *limited* access to debt. Moreover, the behaviour of investors is determined by the transformation of probabilities according to an *informational* parameter. On these grounds, as shown in *Section 2*, the expected return of the guaranteed portfolio, containing a combination of risk and a cash balance, is maximised when the mathematical expectation of the residual exposure (a measure of the cost of assuming bankruptcy) plus the opportunity cost of capital, is minimised. In this way, I follow Dhaene et al. (2003), who on these terms develop a mechanism for capital allocation (see also Goovaerts et al., 2005).

In fact, the individual demands for liquidity are given by the *quantile* functions (or *Value-at-Risk*) of the random variables describing risks in the generalised model, where the arguments are occupied by the net opportunity returns on capital. In this way, risk and expectations explicitly affect the amounts of demanded cash balances. When looking for the aggregated surplus in *Section 3*, capital is supposed to be provided by a central authority or by financial intermediaries acting in a competitive market, in such a way that a single interest rate is required for lending. Hence the situation is similar to the case of a centralised conglomerate distributing capital among subsidiaries (Dhaene et al., 2003; Goovaerts et al., 2005) and the opportunity cost of money is related to the average return over a class of money substitutes. In this context, the money-demand of the economy is also given by a probability quantile, but referred to the riskiness of a *market-portfolio*, determined by the *comonotonic* sum of sectorial exposures — where the *comonotonic* dependence structure characterise the situation when no diversification is possible. As a result, under certain risk-parametrisations, the values of the parameters at the aggregated level are expressed as the weighted average of individual estimations (as proved in *Section 4*) and then the state of aggregation of the economy plays a role as well in the determination and stability of the liquidity demand.

As stated in *Section 4*, depending on the statistical characterisation of the series of capital returns on output, different parametrisations are then allowed for the demand for liquidity. In particular, the case of constant interest-rate elasticity, very popular in the literature of the money-demand, is related to Paretian risks. In *Section 5*, it is shown how the extended model gives a theoretical background for many of recent investigations trying to explain overestimations of the classic constant-elasticity relationship when applied to actual cash-balances observed after 1970. A new framework for the analysis of macroeconomic stability and the conduction of monetary policy is then provided in *Section 6*. The final remarks are given in *Section 7*.

2 Liquidity-Preference as Rational Behaviour

In the model of Tobin (1958), investors hold portfolios producing random capital returns and equity is maintained in order to avoid insolvency — for *precautionary* motives. The markets of securities are supposed to run under *perfect conditions*, in such a way that risks are completely determined by two parameters, expected return and *volatility* (expressed as the standard deviation of the series of returns); lending and borrowing are allowed at any moment for a common risk-free rate of interest r_0, and finally, at any point of time, investors share expectations concerning the future performance of securities and thus portfolios — the *efficient market's hypothesis*. Hence cash-balances can be reinvested to earn the sure return r_0 and their levels can be modified at any moment, for in perfect markets there is always somebody who agrees to take or provide any amount of money at the market price. The percentage returns of portfolios containing combinations of risk and equity can then be expressed in terms of the percentage returns of the underlying funds, in such a way that $Y = (1 - l) \cdot X + l \cdot r_0$, where l denotes the proportion of output maintained as a cash-balance, with $L = \overline{Y} \cdot l$. On these grounds, a linear relationship determines the set of *efficient* portfolios, in the sense that for any combination outside this line, it is always possible to build a new fund providing the same expected return and a lower risk, or the same risk but a higher return.

The *Sharpe Ratio* gives the rate at which investors agree to substitute risk and equity in this framework (Sharpe, 1966). High volatilities offer the chance of large capital profits at the price of equivalent chances of large capital losses, while low volatilities diminish the size of capital losses but offer little prospects of gains. Consequently, a higher return can be obtained only if more risk is assumed. The way preferences affect portfolio decisions can then be analysed in the plane (σ_Y, μ_Y), where the indifference-curves of risk-lovers have negative slope, as long as such individuals accept a lower expected return if there is a chance to obtain additional gains. By contrast, averse-to-risk investors do not take more risk unless they are compensated by a greater expected return and consequently, their indifference-curves have positive slopes. Moreover, *more* is regarded as *better*, so that indifference-curves located to the upper left corner of the plane are related to higher utilities. Therefore, for any risk-aversion profile, the optimal combination is determined by the (tangency point of) intersection between the unique indifference curve representing preferences and the line of efficient portfolios.

On these grounds, Tobin regards liquidity-preference as *behaviour towards uncertainty*. The question naturally arises whether this result is maintained when the hypothesis of the model are relaxed — in particular, when frictions in credit markets impose restrictions on liquidity. A *generalised* or *extended* model will be developed in the following to characterise preferred cash-balances in *imperfect* capital markets, in such a way that (not surprisingly), risks are taken from a general class of probability distributions that economic agents distort according to their information and knowledge; investors are supposed to face liquidity constraints at borrowing and lending; information is not fairly distributed and managers have to expend effort to correctly asses prices — and consequently, they keep different expectations about risks. Under such circumstances, the guarantee L maintained for precautionary purposes cannot be invested on a current account to obtain the sure return r_0, for non-risky bonds cannot be converted to money at any moment without incurring in additional costs in imperfect markets — money and risk-free securities are not perfect substitutes any more. The percentage capital return of the *guaranteed* or *net-* portfolio, containing the fund X and the guarantee $L = \overline{Y} \cdot l$, is then expressed as $Y = X - l - r_0 \cdot l$.

The expected value of the percentage return Y can be determined by analysing the different payments at maturity. Thus, when $X > l$, capital profits (over the level of risk-capital) are obtained whose size is given by the term $\overline{Y} \cdot (X - l)_+$. Such a surplus can be used to pay current liabilities or assigned to new investments. By contrast, when $X < -l$, bankruptcy is declared and the amount L is given to creditors, who have to afford the residual amount $\overline{Y} \cdot (X + l)_-$. Alternatively, investors can attract funds in the market to avoid default. In any intermediate case, i.e. when $-l \leq X \leq l$, no capital gains are attained, though capital losses are bounded by the size of the guarantee. The fund should be rebuilt or sold in this situation and so its value is equal to zero. Letting the parameter θ denote the state of information of the decision-maker, we obtain that the expected return of the net-portfolio is given by:

$$\mu_{\theta,Y} = E_\theta \left[\, (X - l)_+ \right] - E_\theta \left[\, (X + l)_- \right] - r_0 \cdot l = \Delta(l) - r_0 \cdot l \qquad (2)$$

The term $\Delta(l) := E_\theta \left[\, (X - l)_+ \right] - E_\theta \left[\, (X + l)_- \right]$ represents the *economic margin* obtained because of financial intermediation, while $E_\theta \left[\, (X + l)_- \right]$ accounts for the *cost of assuming bankruptcy* — a role that can be adopted by the own investor, an insurance company or a central authority.

The *informational* parameter θ is supposed to be determined by aversion-to-risk, but also by *information* and *knowledge*, in the sense that (as stated by De Finetti, 1975) *information* *and knowledge permit a limitation of expectations.* Consequently, let us define:[1]

$$\mathrm{E}_\theta\left[X\right] = \int x \; dF_{\theta,X}(x) = \int T_{\theta,X}(x) \; dx := \int T_X(x)^{\frac{1}{\theta}} \; dx$$

The *cumulative* and *tail-* (also known as *decumulative* or *survival*) probability distribution functions, respectively denoted by $F_{\theta,X}$ and $T_{\theta,X}$, have been introduced, $F_{\theta,X}(x) = P_\theta\left[X \leq x\right] = 1 - P_\theta\left[X > x\right] = 1 - T_{\theta,X}(x)$. When $\theta > 1$ the expected value of risk is overestimated and underestimated when $\theta < 1$, in this way respectively accounting for the behaviour of *averse-to-risk* and *risk-lover* investors. The *rational* demand for money is thus determined by the first order condition:[2]

$$\frac{\partial}{\partial l} \; E_\theta\left[\; (X - l)_+\right] - \frac{\partial}{\partial l} \; E_\theta\left[\; (X + l)_-\right] - r_0 = -T_{\theta,X}\left(l^*\right) + F_{\theta,X}\left(-l^*\right) - r_0 = 0$$

Given that $F_{\theta,X}\left(-l\right) = P_\theta\left[\; X \leq -l \;\right] = P_\theta\left[\; -X > l \;\right] = T_{\theta,-X}\left(l\right)$, the following equivalent characterisation is obtained:

$$T_{\theta,-X}\left(l^*\right) - T_{\theta,X}\left(l^*\right) = r_0 \tag{3}$$

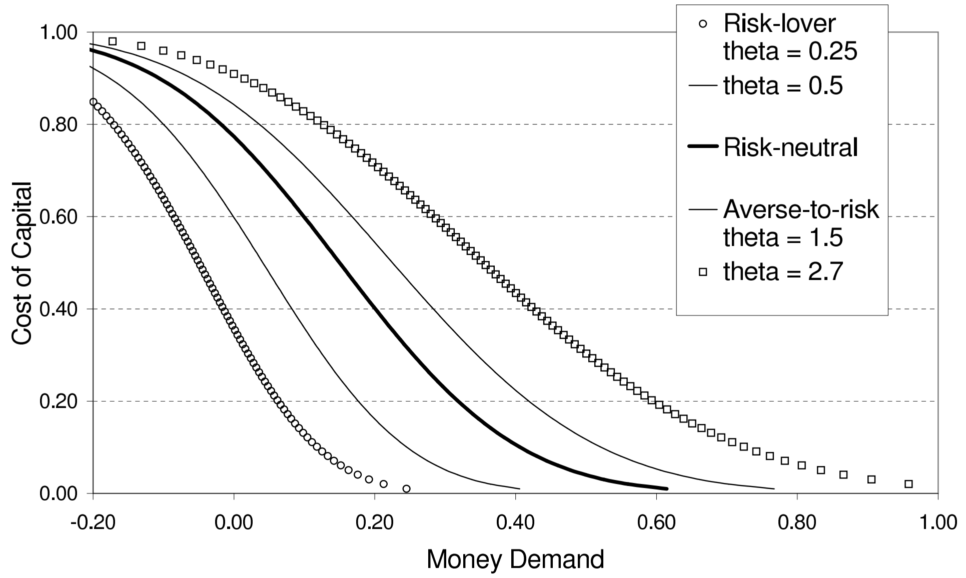
Hence the rational money-demand is determined at the point where the instantaneous benefit from liquidity, given by the difference of the probabilities of the events when the capital return of the insured portfolio is less and greater than zero, equals the marginal return of the sure investment. In this context, the optimal capital allocation involves an optimal exchange of a sure return and a flow of probability and it is the mass accumulated in the *tails* of the distribution-function what matters. Such a conclusion stresses the role of money as a *capital asset*. Moreover, it is possible to assure mathematically that the optimal level l^* exists as long as the expected return on income $\Delta(l) - r_0 \cdot l$ is a non-decreasing and concave function, i.e. if $\Delta'(l) > r_0$ and $\Delta''(l) < 0$ or equivalently, if $T_{\theta,-X}\left(l\right) - T_{\theta,X}\left(l\right) > r_0$ and $T'_{\theta,-X}\left(l\right) - T'_{\theta,X}\left(l\right) < 0$. The first inequality implies that the marginal benefit of risk-capital is greater than its opportunity cost — and accordingly incentives are maintained to demand it. The second condition ensures concavity. Although no explicit relationship is provided in *Equation 3* for the cash-demand, a numerical procedure could be implemented to find the solution.

As long as capital gains are reinvested on the assets contained in the mutual fund X, it is the average return offered by this class of securities which determines capital decisions.

[1] The *proportional hazards distortion* is introduced (Wang, 1995), so called since it is obtained by imposing a safety margin to the *hazard rate* of risk in a multiplicative fashion: $h_{\theta,X}(x) = \frac{1}{\theta} \cdot h_X(x) := \frac{1}{\theta} \cdot \frac{d}{dx} \; ln \; T_X(x)$, with $\theta > 0$. Other distortions can be used instead. In the general case, a *distortion function* is defined (over the unit interval) and an axiomatic description is provided for the distorted price (Wang & Young, 1998). *Averse-to-risk* and *risk-lovers* investors are then respectively characterised by *concave* and *convex* transformations. All the analysis that follows is maintained in the same terms under this general setting (see also Mierzejewski, 2006).

[2] Applying the Leibnitz rule, such that $\frac{d}{dl} \int_{u(l)}^{v(l)} H(l,x) \; dF_{\theta,X}(x) = \int_{u(l)}^{v(l)} \frac{d}{dl} H(l,x) \; dF_{\theta,X}(x) + H\left(l, v(l)\right) \cdot v'(l) - H\left(l, u(l)\right) \cdot u'(l)$ (see, for example, Churchill, 1958) the relationship is obtained by noticing that: $E_\theta\left[\; (X - l)_+\right] = \int_l^\infty (x - l) \; dF_{\theta,X}(x)$ and $E_\theta\left[\; (X + l)_-\right] = -\int_{-\infty}^{-l} (x + l) dF_{\theta,X}(x)$.

Figure 1: Demanded cash-balances for Gaussian risks and different distortions.



Additionally, the *Mean-Value Theorem* assures that a rate of return $0 < r_{\theta,X} < 1$ exists that satisfies:

$$E_\theta \left[\, (X - l)_+ \right] \;\approx\; E_\theta \left[X_+ \right] - r_{\theta,X} \cdot l \tag{4}$$

Since the coefficient $r_{\theta,X}$ represents the marginal reduction in insured capital gains produced when attracting the next unit of equity, it can be regarded as a *premum for solvency*. From *Equations 3* and *4*, the following expression is obtained for the expected percentage income:

$$\mu_{\theta,Y} = E_\theta \left[X_+ \right] - E_\theta \left[\, (X + l)_- \right] - (r_0 + r_{\theta,X}) \cdot l$$

Under such conditions, *precautionary* investors that maximise value minimise bankruptcy costs. Applying Lagrange optimisation, we obtain that decision makers attracts funds until the marginal return of risk equals the total cost of capital:

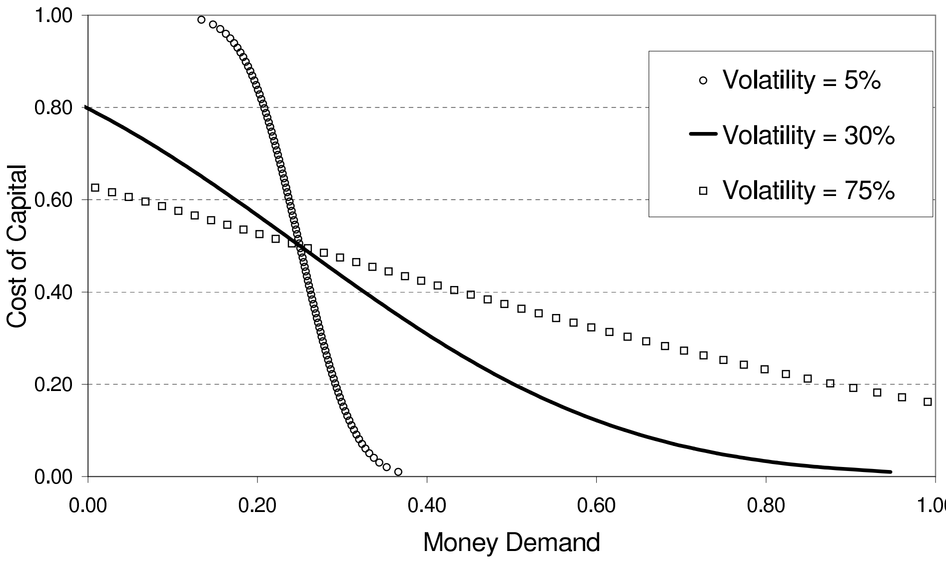
$$-\frac{\partial}{\partial l} \, E_\theta \left[\, (X + l)_- \right] - (r_0 + r_{\theta,X}) = T_{\theta,-X} \left(l^* \right) - (r_0 + r_{\theta,X}) = 0$$

Equivalently, it can be said that investors stop demanding money at the level at which the marginal expected gain in solvency equals its opportunity cost. Thus the optimal cash demand is given by:

$$l_{\theta,X} \left(r_0 + r_{\theta,X} \right) \;=\; T^{-1}_{\theta,-X} \left(r_0 + r_{\theta,X} \right) \tag{5}$$

From this expression, the money demand follows a decreasing and — as long as the distribution function describing uncertainty is continuous — continuous path, whatever the kind of risks and distortions (see *Figures 1* and *2*). The minimum and maximum levels of surplus are respectively demanded when $(r_0 + r_{\theta,X}) \geq 1$ and $(r_0 + r_{\theta,X}) \leq 0$. Moreover, as depicted in *Figure 1*, averse-to-risk investors (characterised by $\theta > 1$) underestimate the cost of money and consequently prefer to keep a higher surplus. Risk-lovers (characterised by $\theta < 1$) behave in the opposite direction.

Figure 2: Demanded cash-balances for Gaussian risks and different volatilities.



The return $r_0 + r_{\theta,X}$ can be interpreted as a *net* opportunity cost, dependent on the attitude towards risk, information and knowledge of decision-makers, as well as on the perception of the credit quality of firms and operational and administrative costs (both determinants of the processes of capital returns). Though such factors are expected to evolute on time, we can regard them as softly modified — and not a matter of *speculation*. Therefore, the parameter θ is expected to remain stable and consequently, as long as the probability distribution of the random variable X is also stable, the capital decisions of investors should remain more or less the same and the economy as a whole should behave accordingly. However, if probability distributions evolve on time, so does the premium for solvency $r_{\theta,X}$. In fact, this can be the case after a monetary expansion (which can be performed by the central bank as well as by the entrance of new investors) if part of the extra money is used to buy financial securities and the increment in demand is high and persistent enough to induce the term $E_\theta\left[(X - l)_+\right]$ to increase. In a similar way, a monetary contraction can press the premium for solvency to decrease. This situation might in turn impel decision makers to actualise expectations and so the informational parameter θ might be modified. But this adjustment is supposed to be produced with a certain delay — for a time is required for analysis — while the opportunity cost may be *instantaneously* altered. In other words, prices are supposed to adjust more rapidly than quantities (as in Friedman, 1970). By means of this mechanism, changes in the stock of money may induce instability *from within* in secondary markets.

Adjustments are performed along a stable money demand relationship, though the process may be reinforced by structural modifications once expectations are actualised.

3 Liquidity Shocks in Capital Markets

A particular feature of the extended model presented in the previous section is that a *solvency premium*, equal to the marginal reduction in capital gains resulted from attracting the next unit of capital, affects the opportunity cost of money. This means that interest-rates are corrected into conformity with *expectations* and the *prospects* of risks. However, as long as firms can attract funds at the same cost, the adjustment is performed in accordance to a *common* set of information θ, representing the *average* knowledge of the market:

$$r \;=\; r_0 + r_{\theta,X} \tag{6}$$

Already Keynes (1935), on the grounds of the speculative motive, claimed the cash demand depends on the return expected to prevail over a longer period, regarded as an *anticipated* interest rate. This fact is stressed by monetarists (Friedman, 1970). Though r cannot be interpreted as an *anticipated* value in the same terms, it is also affected by *expected* capital gains. Alternatively, the cost of money can be related to the return offered by a class of money-substitutes, as established in the model for the pricing of capital assets of Sharpe (1966) and also by Modigliani and Miller (1958). Within this framework, the aggregated exposure X can be regarded as the percentage return offered by the *market-portfolio* — and determined by the series of returns of a representative index. It then follows that capital can be only afforded by those investors expecting profits over the market rate of interest, i.e. such that $r < r_0 + r_{\theta,X}$, since a net loss is suffered from borrowing when $r > r_0 + r_{\theta,X}$.

In order to obtain an expression for the cash balance demanded by the whole economy, let us assume that economic agents hold aggregate exposures characterised by the random variables $X_1, \cdots, X_n$. Capital is supplied by a central authority at a single interest rate r (or, equivalently, secondary markets are regarded as competitive and financial intermediaries are *price takers*) relying on the informational parameter θ and the uncertainty introduced by the *market* portfolio X. Then the aggregated money demand is given by:

$$l_{\theta,-X}(r) \;=\; \sum_{i=1}^{n} T^{-1}_{\theta,-X_i}(r) \;=\; T^{-1}_{\theta,-X}(r)$$

The second equality is a mathematical identity as long as the process of capital gains and losses of the market portfolio is described by the *comonotonic sum* $X = X_1^c + \cdots + X_n^c$, where $(X_1^c, \ldots, X_n^c)$ represents the *comonotonic random vector* with same marginal distributions as $(X_1, \ldots, X_n)$ and *comonotonicity* characterises an extreme case of dependence, when no benefit can be obtained from diversification[3]. Thus precautionary investors rely on the *most pessimistic* case, when the failure in any single firm spreads all over the market. When differing expectations are allowed among decision makers, the aggregate money demand is given by:

[3]The inverse probability distribution of the comonotonic sum is given by the sum of the inverse marginal distributions (Dhaene et al., 2002).

$$l_{\theta_1,\ldots,\theta_n,-X}(r) \;=\; \sum_{i=1}^{n} T^{-1}_{\theta_i,-X_i}(r) \;=\; T^{-1}_{\theta_1,\ldots,\theta_n,-X}(r)$$

where $T_{\theta_1,\ldots,\,\theta_n,-X} = \left(\sum_{i=1}^{n} T^{-1}_{\theta_i,-X_i}\right)^{-1}$ denotes the distribution function of the comonotonic sum when marginal distributions are given by $(T_{\theta_1,-X_1},\ldots,T_{\theta_n,-X_n})$.

Let M denote the total offer of money in the economy and let us analise how the monetary equilibrium is established in the short-run:

$$M \;=\; \overline{Y}\cdot l_{\theta_1,\ldots,\theta_n,-X}\,(r)$$

As long as the expectations of decision makers and the stochastic nature of risks remain unchanged, the cost of capital r and the level of income $\overline{Y}$ are forced to vary for the liquidity-demand to fit a given stock of money. Hence, to avoid undesired fluctuations, the central bank can modify the rate of interest to adapt the liquidity-preferences of investors to a new equilibrium while keeping stable the level of income. However, as explained in *Section 2*, when the processes of assets' returns are not stationary, part of the adjustment might be prevented by the premium for solvency. This means that a stronger or a weaker movement of the interest rate may be needed to reestablish the monetary equilibrium.

The mechanism can be regarded as an adjustment to *inflationary* and *deflationary* trends in capital markets. In fact, if part of the cash available after a monetary expansion is invested on assets, the total transactions spending, expressed in nominal units, will increase, thus putting a pressure on prices to rise — though not necessarily affecting the *real* side of the economy. As a consequence, the probability of obtaining capital gains will increase and so will the premium for solvency, in this way stimulating decision makers to keep fewer provisions. By contrast, after a monetary contraction, security prices might fall as a consequence of the contraction of demand, independently of the *real* prospects of investments.

As times pass by, liquidity preference is affected by the new stochastic regime of the market portfolio (characterised by the random variable X) and so the interest-rate-elasticity of the money demand may change. Moreover, individual exposures will be more or less altered depending on the variations in the amounts of money spent on the respective assets. In the medium-term investors actualise their expectations according to the new market conditions. Since different combinations of the informational parameters $\theta_1,\ldots,\theta_n$ may lead to the same cash balance, the model allows multiple equilibria. Therefore, even if the authority succeeded in stabilising the level of income, the new equilibrium may involve a different distribution of resources in the economy.

4 Alternative Parametrisations for the Money-Demand

If the series of returns of national output is represented by a Gaussian random variable with mean μ and volatility σ, the tail-probability-function is given by $T_{-X}(x) = 1 - F_{-X}(x) = 1 - \Phi\left(\frac{x-\mu}{\sigma}\right)$, where Φ denotes the cumulative probability-function of a *standard* normal distribution (see Dhaene et al., 2002). Notice that, since the estimations of the risk-parameters μ and σ are based on the outcomes of the series of capital *losses* $(-X)$, positive and negative values of the expected return are respectively related to the cases when a capital loss and a capital profit is obtained. Replacing in *Equation 5* we obtain that the amount of capital demanded by a rational decision-maker is determined in the following way:

$$l_{\mu,\sigma}(r) = \mu + \sigma\ \Phi^{-1}(1-r) \tag{7}$$

Hence the size of preferred cash-balances increases with the magnitude of expected capital losses when $\mu > 0$ — while it decreases with the size of expected profits when $\mu < 0$. Within a class of securities showing the same expected return, higher volatilities are related to less efficient funds — for the same mean return is obtained in this case at the cost of assuming more risk. Accordingly, at any level of the rate of interest, the demand for money increases with volatility, which means that it is more elastic with respect to its opportunity cost when the riskiness of macroeconomic balances is incremented (as depicted in *Figure 2*). Moreover, as a consequence of the symmetry of the Gaussian distribution, the demand curves intersect at the point $r = 0.5$. At this level, there is an equal chance of obtaining capital profits and losses, no matter the volatility of national income, for $\Phi^{-1}(0.5) = 0$, and accordingly the same amount of cash is demanded — equal in magnitude to the expected return of the fund.

The non-linearity of the standard Gaussian distribution is inherited by the money-demand, though a linear relationship seems to provide a good approximation across a long range of the rate of interest (see *Figures 1* and *2*). Since the cumulative probability-distribution can be related to the *density* function $\phi(x) = (2\pi)^{-\frac{1}{2}} \cdot \exp\left(-\frac{x^2}{2}\right)$, in such a way that, for any probability-level p:

$$\frac{\partial \Phi(x)}{\partial x} = \phi(x) \quad \Longrightarrow \quad \frac{\partial \Phi^{-1}(p)}{\partial p} = \frac{1}{\phi\left(\Phi^{-1}(p)\right)} \quad , \quad \textit{with} \ \ p = \Phi(x)$$

the following expression is obtained from *Equation 7* for the *point interest-rate-elasticity* of the money-demand:

$$\epsilon_r^{GA}(r) \ = \ \frac{r}{l_{\mu,\sigma}(r)} \cdot \frac{dl_{\mu,\sigma}(r)}{dr} \ = \ \frac{-(2\pi)^{\frac{1}{2}}\ \sigma\ r}{\mu + \sigma\ \Phi^{-1}(1-r)} \cdot \exp\left(\frac{\left[\Phi^{-1}(1-r)\right]^2}{2}\right) \tag{8}$$

Moreover, since $\Phi^{-1}(1-r) \to +\infty$ when $r \downarrow 0$ and $\Phi^{-1}(1-r) \to -\infty$ when $r \uparrow 1$, it is possible to prove that:

$$|\epsilon_r^{GA}(r)| \to \infty \quad \textit{when} \ \ r \downarrow 0 \ \ \textit{and} \ \ r \uparrow 1$$

Hence the money-demand turns more elastic with respect to the rate of interest as the latter approaches to the values $r = 0$ and $r = 1$. At those levels, the economy is found in a *liquidity-trap*, since in this case no amount of capital can satisfy the preference for liquidity of decision-makers. The same conclusion can be attained by a purely mathematical reasoning in the extended model, for as long as capital decisions are determined by the exchange between a sure return and a flow of probability, the cost of capital is related to a confidence probability-level. Thus, when $r = 1$ capital profits of *infinite* magnitude are expected with probability one, i.e. they are produced with certainty and accordingly, investors not only do not keep provisions, they actually agree to take additional liabilities to funding investment projects. By contrast, when $r = 0$ capital losses of *infinite* magnitude are expected with certainty and so investors never feel they are properly insured, whatever the size of provisions.

When the cash-balance demanded by the economy is supposed to be determined by the capital requirements produced at different scales (with levels of aggregation varying from individual investors and small companies to financial conglomerates and economic sectors), the national liquidity-preference can be obtained by adding the surpluses maintained for individual risks, as stated in *Section 3.* Let us assume, accordingly, that individual risks are distributed as Gaussians with means $\mu_1, \ldots, \mu_n$ and volatilities $\sigma_1, \ldots, \sigma_n$, which are expressed as proportions of the levels of income and can be interpreted as the volatilities of different funds as well as the distorted volatilities of a same Gaussian exposure — or some intermediate case. The contributions of individual exposures are given by the coefficients $\lambda_1, \ldots, \lambda_n$, with $0 \leq \lambda_i \leq 1 \; \forall i$, in such a way that $\overline{Y}_i = \lambda_i \cdot \overline{Y}$ and $\overline{Y} = \overline{Y}_1 + \cdots + \overline{Y}_n$. Then the capital return of the market-portfolio is represented by the comonotonic sum of individual exposures, which is also a Gaussian random variable whose mean and volatility are respectively given by (see Dhaene et al., 2002):

$$\mu \;=\; \sum_{i=1}^{n} \lambda_i \cdot \mu_i \qquad \& \qquad \sigma \;=\; \sum_{i=1}^{n} \lambda_i \cdot \sigma_i \tag{9}$$

Therefore, the mean return and the volatility of the series of income's returns are determined by the weighted average of decision-makers' estimations. In this context, the liquidity-demand of the economy could be mostly explained by the conditions in a specific predominant sector. The deep relationship between financial markets and the markets of goods and services is stressed in this way. As long as the behaviour of the economy at the macroscopic level is obtained by the average of both sectors, instability can be transmitted from one to another, depending on the riskiness in each of them and the relative importance in the economy with respect to each other.

It is traditionally assumed in empirical studies of the money-demand that its interest-rate elasticity does not depend on the level of the rate of interest. Such specification is attained in the extended model when capital returns are *Paretian distributed*, a case where tail-probabilities depend on a single parameter $\alpha > 0$ in such a way that $T_{-X}(x) = x^{-\frac{1}{\alpha}}$, with $x > 1$, so that liquidity-preference is determined by $l_\alpha(r) = r^{-\alpha}$, with $0 \leq r \leq 1$. As long as investors agree on a single informational state α, the comonotonic sum of individual cash-balances is also Paretian distributed (Dhaene et al., 2002) and the agregated money-demand is given by:

$$l_{n,\alpha}(r) = n \cdot r^{-\alpha} \tag{10}$$

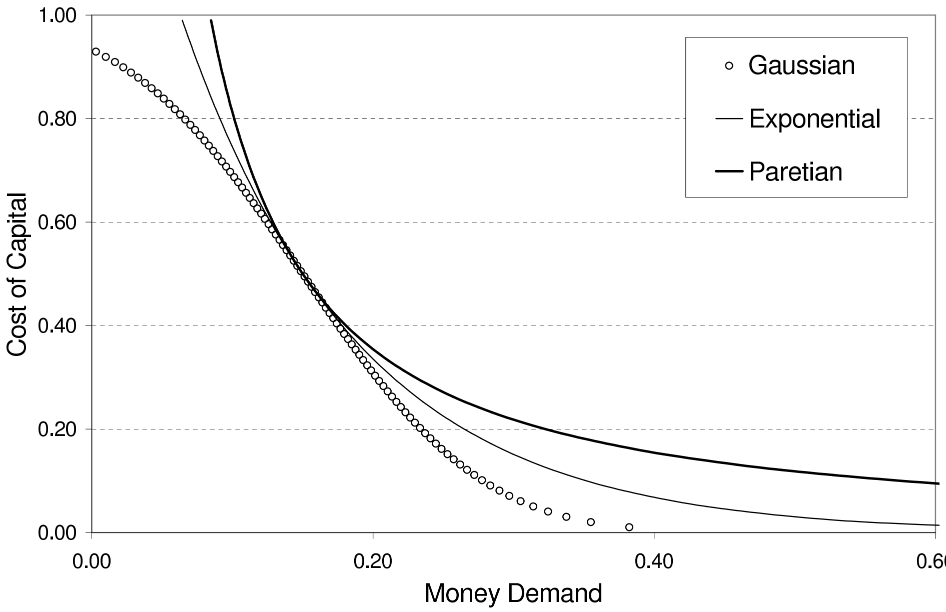
Under differing expectations, the comonotonic sum is not necessarily Pareto-distributed, although estimations of the parameters n and α can be found for the liquidity-demand to be determined by *Equation 10.* In this context, the intercept obtained by performing a statistical regression on logarithmic variables can be related to the logarithm of the number of economic sectors, i.e. $intercept = \ln(n)$, and can thus be regarded as a measure of *aggregation.*

It can be easily proved that the point interest-rate-elasticity is constant under a Paretian specification and its magnitude is equal to the exponent α:

$$\epsilon_r^{PA}(r) \;=\; \frac{r}{l_{n,\alpha}(r)} \cdot \frac{\partial l_{n,\alpha}(r)}{\partial r} \;=\; \frac{r}{l_{n,\alpha}(r)} \cdot \left[-\alpha \; n \cdot r^{-\alpha-1}\right] \;=\; -\alpha \quad \forall \; r \tag{11}$$

From the mathematical point of view, the greater the parameter α, i.e. the smaller the ratio $\frac{1}{\alpha}$, the more the probability accumulated in the tails of the distribution-function. In particular, when $1 \leq \frac{1}{\alpha} < 2$, the second-order statistical moment or *variance* does not exist (i.e. it is equal to infinity), while even the first-order statistical moment or *mean-value* of the distribution-function does not exist when $\frac{1}{\alpha} < 1$. On these grounds, the *riskiness* of Pareto-distributed random variables is determined by the exponent α and then, as in the Gaussian setting, the magnitude of the interest-rate elasticity increases with the exposure to risk.

Figure 3: Demanded cash-balances for different risk-specifications.



Still an alternative representation can be obtained by considering that individual exposures are *exponentially distributed*. The tail-probability depends on the pair of parameters (C, β) in this case, in such a way that $T_{-X}(x) = \exp\left(\frac{C \,-\, x}{\beta}\right)$ and the liquidity preference function of the economy is given by:

$$l_{C,\beta}(r) = C - \beta \cdot ln(r) \quad , \quad with \quad \beta > 0 \tag{12}$$

In this case, the comonotonic sum is also exponentially distributed (see Dhaene et al., 2002), such that if $C_1, \ldots, C_n$ and $\beta_1, \ldots, \beta_n$ denote the informational types of investors, with $\beta_i \geq 0 \; \forall i$, and, as before, $\lambda_1, \ldots, \lambda_n$ represent the marginal contributions to the aggregated income, with $0 \leq \lambda_i \leq 1 \; \forall i$, then the risk-parameters of the market-portfolio are expressed as the weighted average of the exponential parameters of marginal risks:

$$C \;=\; \sum_{i=1}^{n} \lambda_i \cdot C_i \qquad \& \qquad \beta \;=\; \sum_{i=1}^{n} \lambda_i \cdot \beta_i \tag{13}$$

The interest-rate elasticity of the money-demand of the economy is then given by:

$$\epsilon_r^{EX}(r) \;=\; \frac{r}{l_{C,\beta}(r)} \cdot \frac{\partial l_{C,\beta}(r)}{\partial r} \;=\; \frac{r}{C - \beta \cdot \ln(r)} \cdot \left[\frac{-\beta}{r}\right] \;=\; \frac{-\beta}{C - \beta \cdot \ln(r)} \tag{14}$$

Thus, ceteris paribus, the lower the parameter C or the higher the exponent β, the more sensitive is liquidity-preference to the cost of capital. In this way, as in the Gaussian setting, the variability of output is explicitly related to the elasticity of the money-demand with respect to the rate of interest.

In *Figure 3*, the money-demands related to the different risk-parametrisations analysed in this section are depicted. They are *equivalent* representations of the optimal surplus for the same market-portfolio, in the sense that the interest-rate elasticity and the preferred cash-balance are equal for the three curves at the point $r = 0.5$ — in this way, the curves are closest to each other. It can be seen that, at any level of interest-rates, the amount of capital preferred under a Paretian setting is greater than under the exponential, which in turn is greater than under a Gaussian framework. Such a conclusion is consistent with the fact that the probability accumulated in the tails of Paretian probability-distributions is always higher than the probability accumulated in the tails of exponential distributions, and exponential tails are fatter than Gaussians. Accordingly, Paretian claims are *riskier* than exponential which are *riskier* than Gaussian. In this way, the result obtained within risk-classes, that is, that the riskiness of output is related to the elasticity of the money-demand with respect to the cost of capital, is valid as well when the preferences for liquidity corresponding to different families of risks are compared.

5 Classic and Recent Trends in the Investigation of the Money-Demand

On the grounds of *Equation 1*, it is customary to relate the logarithms of real cash-balances to the logarithms of real income and interest-rates, in such a way that if the subscripts t denote the measures at different points in time:

$$\ln\left(\frac{M_t}{P_t}\right) = M_0 + \epsilon_y \cdot \ln(y_t) + \epsilon_r \cdot \ln(r_t) + \Gamma_t \quad \textit{with } t > 1 \tag{15}$$

Lags in the series of the real stocks of money $\frac{M_t}{P_t}$ are frequently introduced. The term Γ_t represents a stochastic perturbation (usually assumed to be a Gaussian random variable) accounting for deviations from the basic trend when replacing empirical observations. The coefficients ϵ_y and ϵ_r respectively denote the point-elasticities with respect to real income and interest-rates. In fact, such specification corresponds to a generalisation of *Equation 1*, where $\epsilon_y = 1$ is chosen.

The evidence emerged prior to 1974 suggested that such relationship adequately explained the quarterly movements of the money-demand. However, when extrapolating to the data gathered after that year, the model was found to significantly over-predict actual cash-balances. The classic reference on this case is Goldfeld (1976), who relates such over-predictions to the downturn phase of the economic cycle observed in that period — which shows the most severe recession of the postwar era, accompanied by double-digit inflation and high interest-rates, as well as many institutional changes in the financial structure. By contrast, the regression performs well in upturn phases. The question established by Goldfeld was then whether the deviations to be seen under extreme conditions can be explained by a reassessment of the model. Let us examine in the following if a theoretical explanation can be provided on the light of the generalised model proposed in this paper.

As already stated in *Section 4*, the models of *Equations 10* and *15* correspond to equivalent characterisations of the money-demand when the randomness of national income is statistically described by a Paretian distribution. Since then $M_0 = \ln(n)$, where n denotes the number of investors, the intercept can be regarded as determined by population — and hence inflationary and deflationary trends can be stimulated by changes in the size and the composition of market's participants. Moreover, the interest-rate elasticity is related to the exponent of the probability-function, which in turn measures the exposure to risk — and hence the dependence of the money-demand on risk is explicit in the generalised model. Such a view departs from traditional attempts to incorporate uncertainty as a determinant of the money-demand, where volatility is usually introduced as an additional regressor — without providing any theoretical reasoning supporting this relationship. In these models, riskiness is explained by macroeconomic uncertainty (as in Greiber and Lemke, 2005) or variability in equity markets (as in Carpenter and Langue, 2002). In both cases, risk's proxies prove to be statistically significant over the periods under study.

Choi and Oh (2003) suggest an approach similar to the one presented in this paper and derive a money-demand function from a general equilibrium framework. In so doing, they suppose that monetary and output variables are described by log-normal processes, while investors preferences are given by a discounted expected-utility operator. Then, in the lines of Tobin (1958), liquidity-preference is determined by the maximisation of utility, though certain constraints representing budget and market conditions are also considered. In this context, for any level of nominal interest-rates, cash-balances are negatively and positively affected by output and monetary uncertainty respectively. The model allows to satisfactorily explain departures of observed stocks of money and interest-rates from *Equation 15* in the early nineties (see also Atta-Mensah, 2004).

A different line of argumentation stresses the fact that the line separating the roles of cash and financial assets has blurred due to the innovations and regulatory reforms performed since 1980. In fact, while some instruments bearing interest are currently used for transactions, some cash-deposits bear interest. In this context, the *velocity shocks* occurred after 1980 could be explained by increased volatility in near-money returns and could thus be interpreted as movements along a (stable) money-demand curve. In fact, as noticed by Teles and Zhou (2005), although nominal interest rates have decreased considerably in the last quarter century, the monetary aggregate $M1$ has increased very little in the same period (see also Ball, 2001). A comparison between the evolution of the different monetary aggregates $M1$, $M2$, $M3$ and MZM (see *Figure 4* in Teles and Zhou, 2005) shows that $M1$ has indeed grown at a lower rate than other monetary aggregates since 1980 and has remained roughly constant after 1994 (thus, for example, $M1$ and MZM have respectively incremented in a five and nine percent since 1980).

On these grounds, departures from the long-term relationship of *Equation 15* can be suppressed or at least diminished as long as the returns offered by the *right* monetary aggregate are considered. Thus, based on a stochastic model of general equilibrium, Teles and Zhou (2005) obtain a relationship for the money-demand depending on the differences between the returns offered by a class of nominal bonds and a class of money substitutes. They estimate the model for the measures $M1$ and MZM and find that changing the monetary aggregate measure preserves the long-run relationship of the money demand up to a constant factor. Alternatively, Ball (2001) propose a *co-integrating* relationship for monetary aggregates, with real income and interest-rates as explanatory variables and a stationary error-term. *Non-co-integration* is then rejected when sufficient lags of the variables are introduced and deviations from the *long-run state* are smaller when the aggregate MZM is considered instead of $M1$. A similar model is proposed by Dreger and Wolters (2006), but using $M3$ and introducing inflation as an additional explanatory variable.

In conclusion, overestimations of demanded cash-balances are possible to be found in the extended model as long as the variability of output is overestimated (as in *Figure 2*) or the risk-specification is not correct (as in *Figure 3*). Moreover, since the *private* estimations of the cost of capital depend on the solvency-premiums paid by the underlying funds, which are determined by the probability accumulated in the tails of the series of capital gains (see *Equations 4* and *6*), not only the choice of the monetary aggregate and the variability of the corresponding return determine the optimal level of equity, but also the expected magnitude of the insured gain. Accordingly, during upturn and downturn cycles of the economy higher and lower flows of capital are respectively observed, for the premium for solvency is higher in the former than in the later case — recall that only those firms whose private estimations of the cost of capital are higher than the market return can afford cash-balances. In this way, the roles of the monetary aggregate and riskiness in the determination of the money-demand are incorporated in a single model.

Finally, another determinant of the money-demand recently proposed is the state of aggregation of the industry. Thus, for example, Calza and Sousa (2003) have pointed out that the lack of synchronisation of shocks to national money-demands implies that a more stable environment is guaranteed when nations are gathered into communities. This argument is given to explain, in particular, why the money-demand has been more stable in the euro area than in other large economies. But most importantly, the fact that Germany has a large weight in the $M3$ aggregate for the euro area and that the money demand has been historically stable in that country contributes to the overall stability of the euro area money demand. In other words, the stability of the German economy is shared by the rest of the economies in the block, as a positive externality. Such a behaviour has a theoretical support in the generalised model, for the risk-parameters of the economy are expressed as the weighted average of individual estimations under certain specifications (as established in *Equations 9* and *13*).

6 Monetary Policy and Macroeconomic Stability

Taking first differences to the logarithmic variables of *Equation 1* we obtain that percentage changes in the stock of money can be expressed in the following way:

$$\frac{\Delta M}{M} \;=\; \frac{\Delta P}{P} + \frac{\Delta y}{y} + \frac{\Delta l}{l} \;=\; \pi + \frac{\Delta y}{y} + \epsilon_r \cdot \frac{\Delta r}{r} \tag{16}$$

In the long-run, once the growth-rate of output and the preference for liquidity of investors have settled down to trend, a linear relationship is obtained for money growth, inflation and interest-rates variations. Accordingly, as long as the rate of inflation is kept constant (and its changes are known with certainty) and the stock of money is modified by performing open operations in capital markets, the rate of interest will adjust until the new monetary equilibrium is attained. The expected change in the level of the interest-rate can then be estimated from *Equation 16*. The efficacy of the mechanism depends on the magnitude and the *stability* of the interest-rate elasticity of the money-demand. Recall that monetary policy is most effective when the demand for liquidity is inelastic with respect to the rate of interest, and it is useless when the money-demand is perfectly elastic.

Instability is disregarded by monetarists on the grounds of the *rational expectations hypothesis*, according to which, rational investors form their expectations by making the most efficient use of all information provided by past history (see, for example, Modigliani, 1977). Under such conditions, the market prices of firms are equal to the present value of the future cash flows of their net-portfolios (assets minus liabilities). If this were not the case, investors would take advantage of such arbitrage opportunities until capital gains are vanished (see also Modigliani and Miller, 1958). Accordingly, misestimations of actual demanded cash-balances can only be found in short-terms. Keynes (1936) assumes, on the contrary, that liquidity preference is affected by the speculative motive and that economic agents, when forming their expectations, are controlled by *animal spirits*. On these grounds, a big deal of discussion concerning the stability of the liquidity-demand has been dominated by the question of whether investors behave or not *rationally*. If economic agents are *rational*, it is argued, the natural state of capital markets is to be in *equilibrium*, in such a way that any attempt of authorities to reduce undesired fluctuations would be frustrated by investors trying to make profits from arbitrage-opportunities. By contrast, if economic agents are allowed to base expectations on *animal spirits*, capital markets are inefficient (so that prices are not dependent on fundamentals) and fluctuating, and it is the duty of central authorities to intervene to attain a desired equilibrium.

In the model of Tobin (1958), averse-to-risk decision-makers demand equity in order to avoid suffering *unexpected* capital losses produced when holding *risky* assets. In this context, liquidity-preference is regarded as (rational) behaviour towards uncertainty. Since a unique combination of risk and equity maximises the expected profit of portfolios, a unique stock of money guarantees the economy is found in equilibrium. Moreover, as long as the stochastic processes describing risks are *stationary* and the risk-preferences of investors are kept unchanged, the money demand remains *stable* except for short-term fluctuations inherently to the adjustment mechanism of the market when frictions and unnecessary regulations are confronted. In this way, the inconvenience of governmental intervention is stressed. Instability and crises can only occur in response to sudden flows of information produced by informational asymmetries or simply by *panic* and irrational fear.

Therefore, though the money-demand explicitly depends on risk for random output, the previous analysis is maintained as long as the stochastic process describing output is stationary. But this hypothesis can be hardly sustained in practice — in fact, models of varying volatility are frequently used in the banking and insuring industry. Actually, flows of money are likely to stimulate capital markets by incrementing the amount of transactions, in such a way that depending on whether the demand or the supply for securities grows in a higher proportion, market-prices will be push to increase or decrease respectively. In other words, capital profits and losses will be respectively observed more frequently and more scarcely in the former than in the latter case. The statistical description of the processes of capital returns will then change accordingly and hence the preference for liquidity of the economy cannot be regarded as *stable*.

Although the previous reasoning has been already suggested by researchers in the past, the lack of a mathematical framework describing macroeconomic variables under such circumstances has prevented its development. Let us see how the generalised framework proposed in this paper can provide such a framework. Let ρ denote the set of parameters corresponding to the specification of risk of national output, in such a way that $\rho = (\mu, \sigma)$, $\rho = (n, \alpha)$ and $\rho = (C, \beta)$ for the Gaussian, Paretian and exponential cases respectively (see *Equations 7, 10* and *12*). Thus, if $\overline{y}$ denote the level of real output, the short-run monetary equilibrium is described by the following equation:

$$M \;=\; P\,\overline{y} \cdot l_{\rho}(r) \tag{17}$$

Hence variations in the stock of money can simultaneously affect the level of prices, the level of output, the rate of interest and the values of risk-parameters. If prices are flexible, only P and r are expected to change in the first place. Monitoring and analysis induce investors to eventually incorporate the new stochastic regime of capital returns into decision-making and to modify expectations — both determinants of ρ. In this context, not only informational shocks affect liquidity-preference, but also flows of capital — though a *purely* informational shock (not induced by monetary flows) can produce the same effect.

When national income is determined by the microscopic interactions at some level of aggregation (like economic sectors, industries, companies or individual investors), the riskiness of the market-portfolio is expressed as the weighted average of individual exposures (see *Section 4*). Hence low or high volatility might be induced in the whole economy due to the action of a single group. Benefits from stability due to the more availability to credit are then inherited by less efficient institutions when low volatility predominates, while the contraction of the credit supply has to be afforded by more efficient companies (as a negative externality) when high volatility predominates. In this way, the possibility of *contagion* naturally arises in the model. Actually, if the macroeconomic stability was deteriorated by a single predominant group, medium-size and small companies that are *solvent* but that at the same time rely too-much on short-term debt would find difficulties when trying to fund their normal spending. As long as the magnitude of the shortcut on credit increases, more and more firms of this type will eventually go on bankruptcy.

Alternatively, the variability of the market portfolio might remain unchanged at the time that individual exposures are evolving, since multiple configurations of the parameters

$\rho_1, \ldots, \rho_n$ are compatible with the same aggregated level. In this context, macroeconomic *stability* is simultaneously determined by the size and evolution of sectorial and investors' risk-parameters, as well as by the state of aggregation of the market. Hence companies are able to attract capital on terms and quantities depending on their productivities and the conditions on credit markets. Such a process is essentially *dynamic*. In order to determine the flows of capital between industries, a model of *general* equilibrium would be required — a task that is out of the scope of this paper.

Therefore, within the framework of the generalised model, there is not a single pair of levels of money-stock and interest-rates that satisfy the monetary equilibrium. Actually, according to *Equation 17*, a proper monetary policy should consider a combination of M, P, ρ and r compatible with a given level of real output. *Equation 16* can still be used to obtain estimations of the expected movements of interest-rates in the short-run, though the estimations of the interest-rate elasticity should be actualised according to the risk-parameters. In the long-run, once the uncertainty of output has settle down to a steady level, in such a way that the risk-parameters and so the interest-rate elasticity of the money-demand can be regarded as constant, monetary policy can be conducted as usual. A level of riskiness ρ could then be found preserving the monetary equilibrium for given values of M, P, $\overline{y}$ and r, which can be regarded as the *induced* variability. The level of interest rates could then be determined compatible with given levels of inflation and induced market volatility. Small fluctuations around the steady state will be found as long as the flows of money are small. However, big changes in the stock of money can modify the riskiness of output, in this way changing liquidity-preference and the elasticity of the money-demand.

Special attention has to be paid, however, to the cases when the cost of capital approaches the limits one and zero. As pointed out in *Section 4*, when the risk-specification is different from Paretian, less and more cash-balances than extrapolated by models of constant elasticity are respectively demanded for high and low interest-rates, the greater this difference the more unstable the output and the closer to the axis $r = 1$ and $r = 0$. Accordingly, as long as upturn economic cycles can push the cost of capital to increase (by increasing the demand for funds), the reserves of money will be diminished — in this way stimulating more the economy. In a similar way, lowering the level of interest-rates under recessions will become investors more sensible to the cost of capital and eventually stimulates them to demand all available cash balances for precautionary purposes. Under such circumstances, the economy will be found in a *liquidity trap* and any attempt of the monetary authority to stimulate it by lowering the level of the interest rate will only do matters worse.

7 Conclusions

A model is presented in this paper to characterise the demand for liquidity of investors seeking to maximise expected capital profits when the series of income's variations is random and additional debt can be hired only to a limited extent due to frictions in capital markets. Under such circumstances, decision-makers prefer to maintain a stock of money in order to prevent insolvency (i.e. they demand equity for *precautionary* motives) and a level of surplus exists that maximises the value of the portfolio containing such guarantee and the risky aggregated claim. The *rational* money-demand is thus determined by the exchange of a sure return and a flow of probability and expressed in terms of the *quantile* function, a measure of the probability accumulated in the *tail* of the probability distribution and that is also known as *Value-at-Risk* in the literature of risk-management (see *Equations 3* and *5*). The Value-at-Risk answers the question of how much can be lost in the next trading period for a given *confidence* level of probability. On these grounds, the cost of capital is related to a *confidence-level*. An informational parameter, affecting the opportunity cost of money, represents *expectations*, in such a way that averse-to-risk and risk-lover investors respectively under and overestimate the cost of capital and so they respectively demand more and less equity.

Since national output is obtained by adding the contributions of economic sectors, the cash-balance demanded at the aggregated level could be expressed as the sum of the surpluses maintained by firms and householders. The situation of policyholders is then equivalent to that of central managers distributing equity to divisions in a financial conglomerate, where the cost of capital is related to the average return over a class of money-substitutes. Hence the money-demand of the economy is given by the sum of the liquidity-preferences of investors, mathematically characterised by the *comonotonic sum* of individual exposures, and it is thus expressed as a Value-at-Risk but referred to a *market-portfolio* that relies on the most pessimistic case, when no gain can be obtained from diversification. Moreover, for certain parametric descriptions of risks, the risk-parameters at the aggregated level are equal to the weighted average of individual estimations (as in the Gaussian and Exponential settings, see *Equations 9* and *13*). Two important conclusions are obtained after this reasoning. In the first place, the system accepts multiple equilibria, for different combinations of individual exposures lead to the same market-variability, and secondly, the variability of the market-portfolio could be mainly determined by a single predominant institution or sector.

As shown in *Figures 1, 2* and *3*, the rational money-demand follows a decreasing and (as long as the probability-distribution describing risk is continuous) continuous path. Moreover, the slope of the curve increases with the exposure to risk, i.e. liquidity-preference is more sensible to the interest-rate in more risky environments. However, the point-interest-rate elasticity is independent of the level of the interest-rate only when the series of capital returns is Paretian. In other cases, the liquidity-demand becomes more inelastic when the opportunity cost approaches zero. Such a result is a consequence of the fact that in the generalised model the effect of the cost of capital in the determination of the preferred surplus is the same as that of a confidence level. Thus, when the return on money substitutes approaches zero decision-makers react as if the probability of obtaining capital gains was close to zero and accordingly, they are forced to insure their residual exposures using only cash balances. On the other hand, when the opportunity cost approaches one, the probability of obtaining capital losses is close to zero and so fewer provisions are demanded. Therefore, whatever the level of elasticity that predominates over the medium range of the interest rate, its magnitude converges to infinite when the opportunity cost approaches zero (as can be verified in *Equations 8* and *14*).

The following are thus determinants of the money-demand in the generalised model: the market return offered by a class of money-substitutes; the riskiness of the series of capital returns on national output; the premium for solvency to be paid when maintaining a guarantee for the market-portfolio, and finally, the state of aggregation of the economy. As stated in *Section 5*, recent studies have analysed the effect of such factors over the money-demand, founding that they are statistically significant for the data gathered after 1980.

The efficacy of the monetary mechanism can then be studied within this framework. Thus, while a single combination of the stock of money and the rate of interest satisfy the monetary equilibrium for a given trend of inflation (as specified by *Equations 1* and *16*), the values of risk-parameters are also relevant in the extended model. This means, in particular, that the elasticity of the money-demand with respect to the interest-rate increases with risk, in such a way that monetary policy is more effective the more stable the national output. Moreover, for certain risk-specifications, the elasticity depends on the level of interest-rates and it converges to infinite (perfect elasticity) when approaching to the value $r = 0$.

Finally, recall that while positive returns affect the opportunity cost of money and so determine a movement along a stable relationship, the precautionary attitude of decision-makers depends on the series of negative returns and so does the *shape* of the money demand (see *Equation 5*). Thus, as long as part of the funds available in the economy are spent on capital assets, an adjustment in the opportunity cost r is expected in the short-run — stimulated by the modification of the stochastic nature of capital gains — which is supposed to *instantaneously* affect liquidity preference (see *Equations 6* and *4*). In the medium-term, investors correct their volatility estimations and so part of the adjustment may be performed through volatility shocks. An important feature of the mechanism is that volatility changes, motivated by flows of funds, determine expectations and not the opposite, though liquidity preference might also be affected by a *purely* informational shock — not supported by any change on the probability distributions characterising risks.

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