A Multilevel Analysis of Intercompany Claim Counts

Katrien Antonio*† Edward W. Frees ‡ Emiliano A. Valdez §

November 5, 2009

Abstract

It is common for professional associations and regulators to combine the claims experience of several insurers into a database known as an "intercompany" experience data set. In this paper, we analyze data on claim counts provided by the General Insurance Association of Singapore, an organization consisting of most of the general insurers in Singapore. Our data comes from the financial records of automobile insurance policies followed over a period of nine years. Because the source contains a pooled experience of several insurers, we are able to study company effects on claim behavior, an area that has not been systematically addressed in either the insurance or the actuarial literatures.

We analyze this intercompany experience using multilevel models. The multilevel nature of the data is due to: a vehicle is observed over a period of years and is insured by an insurance company under a "fleet" policy. Fleet policies are umbrella-type policies issued to customers whose insurance covers more than a single vehicle. We investigate vehicle, fleet and company effects using various count distribution models (Poisson, negative binomial, zero-inflated and hurdle Poisson). The performance of these various models is compared; we demonstrate how our model can be used to update *a priori* premiums to *a posteriori* premiums, a common practice of experience–rated premium calculations. Through this formal model structure, we provide insights into effects that company–specific practice has on claims experience, even after controlling for vehicle and fleet effects.

Keywords: actuarial science; hierarchical model; multilevel model; experience rating; bonus–malus factors; generalized count distributions.

*Corresponding author: k.antonio@uva.nl (phone: +31 20 525 5020)

†University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands.

‡University of Wisconsin in Madison, USA (jfrees@bus.wisc.edu).

§University of Connecticut, USA (valdez@math.uconn.edu).

1 Introduction

In many countries and for several lines of business, the insurance market is mature and highly competitive. This strong competition induces insurers to classify risks they underwrite in order to mitigate problems of *adverse selection*, resulting from an asymmetric information available between the insurer and the policyholder. To illustrate the nature of adverse selection in automobile insurance, a policyholder's prior driving history is commonly used as a risk rating factor. A person with a poor driving history may look for a company that does not use this rating factor for pricing; use of the rating factor penalizes him or her for past mistakes in the form of higher premiums. Conversely, a person with a good driving history may seek companies that use this rating factor; these companies reward previous good experience with lower premiums. Companies that use a less refined classification system than their competitors tend to attract less desirable risks, which can have a spiraling effect on future claims. Risk classification systems allow insurers to price their products in a fair and equitable manner, and on a sound statistical basis.

Strong competition encourages insurers to utilize detailed classification systems, so refined that they may not have sufficient exposure to produce reliable claims predictions for all risks in the portfolio. To understand their claims distributions, it is common for several insurers to pool their experience, forming a database known as 'intercompany' data. With a database large enough to produce a refined classification system, fair and equitable premiums can be determined more reliably across all risks.

Although insurance companies compete for the same business, economic forces dictate that the loss experience of insurers can differ. During the sales process, insurers use different underwriting standards and pricing structures to attract different mixes of business. After an accident, insurers differ in their procedures to settle a claim, including legal, and the calculations of claims adjustments, thereby realizing different loss experience across companies. Moreover, there are issues of moral hazard, a term used to refer to the tendency of the insured to alter its behavior in the presence of an insurance coverage. Thus, it is possible that an insured with a policy from a particular company may have a different claims experience than if the insured were contracted with another company. For example, some insurers establish premium rating systems that encourage policyholders to avoid reporting minor losses, even if they are contractually covered under the insurance policy.

1.1 Multilevel Modeling

This paper examines an intercompany database using multilevel modeling. Specifically, we consider policy exposure and claims experience data derived from automobile insurance portfolios of a randomly selected sample of ten general (property and casualty) insurance companies in Singapore. Our data comes from the financial records of automobile insurance policies over a period of nine years, 1993-2001.

Multilevel modeling allows us to readily handle individual claims experience and account for clustering at the company level. This paper examines commercial insurance policies by restricting considerations to 'fleet' policies. These are policies issued to customers whose insurance covers at least one vehicle. A typical situation of 'fleet' policies is automobile insurance coverage provided to a taxicab company, where several taxicabs are insured under the same policy. Multilevel models allow us to capture the possible dependence of claims of automobiles within a fleet, a peculiar characteristic of these types of policies. The unit of observation in our analysis is therefore a registered vehicle insured under a fleet policy. Our multilevel model accommodates clustering at four levels: vehicles (v) observed over time (t) that are nested within fleets (f), with policies issued by insurance companies (c).

Ideas of multilevel modeling and inference are now well–developed in the statistics literature (Kreft and deLeeuw (1998), Snijders and Bosker (1999), Raudenbush and Bryk (2002), Goldstein (2003) and Gelman and Hill (2007)). Linear multilevel modeling also has a long history in the actuarial literature, as summarized in Norberg (1986). Norberg credits the idea to Jewell (1975), with early contributions by Taylor (1979) and Sundt (1980). As an example of classic multilevel insurance applications, Sundt briefly mentions (i) insurance claims from a person, with (ii) several people living in a household, (iii) where several houses are in a town, (iv) and many towns in a county and (v) several counties with a country. Norberg (1986) and Frees et al. (1999) discuss the connections between the statistical linear modeling and traditional actuarial literatures.

1.2 Count Data

This paper examines nonlinear models using insurance claim counts. The loss to the insurer usually consists of the frequency component, which refers to the claim count, and the severity component, which refers to the claim amount, given a claim. This paper focuses on the frequency component as historically, it has been believed that most of the variability in the loss comes from this component. The frequency component has been well analyzed in the actuarial literature, at least when cross-sectional and panel data structures are considered. For instance, the modern approach of fitting claims count distribution to longitudinal data can be attributed to the work of Dionne and Vanasse (1989) who applied a random effects Poisson count model to automobile insurance claims. Pinquet (1997) and Pinquet (1998) extended this work, considering severity as well as frequency distributions. Pinquet et al. (2001) and Bolancé et al. (2003) introduced a dynamic element into the latent variable, again using Poisson regression.

Poisson regression is probably the most popular technique for regression with count data. However, recent research in actuarial science (see e.g. Yip and Yau (2005) and Boucher et al. (2007)) has highlighted the use of parametric distributions other than Poisson to accommodate features of insurance count data that are inconsistent with the Poisson distribution. These authors investigated the use of the negative binomial, zero-inflated and hurdle distributions for the analysis of cross-sectional and longitudinal claim counts. Cameron and Trivedi (1998), Winkelmann (2003), Yau et al. (2003) and Lee et al. (2006) discuss similar research in econometrics and biostatistics. Further on, we extend these distributions towards the analysis of multilevel data with more than two levels. In the sequel, we will use the term 'generalized count distributions' to denote the collection of Poisson, negative binomial, zero–inflated and hurdle Poisson distributions.

A data analytic discussion of ratemaking for fleet data has received limited attention in the actuarial literature, Desjardins et al. (2001) and Angers et al. (2006) being the exceptions. They discuss the calculation of bonus–malus factors ('BMF') for a three level data set of claim counts on insured trucks in Québec. They use Poisson regression models with random effects for vehicles and fleets. BMFs are common measures used in the insurance industry to either penalize or reward customers according to their historical claims experience.

1.3 Benefits of Intercompany Data

A multilevel model of intercompany data is of interest to insurance companies, regulators and reinsurers. Insurance companies can use the results of this paper to predict the number of claims not only for each vehicle but also for each fleet. Predictions at the fleet level are particularly important because contracts are written and hence premiums are exchanged for coverage at this level. Further, an insurance company can use a model of several companies to understand and possibly compare their experience with their competitors. To illustrate, given a specific risk class (such as female, aged 20-24 with poor driving experience), is the loss experience for the company high or low compared to the competition? This type of information is extremely useful in a competitive pricing environment.

Regulators and reinsurers typically deal with several companies and so would also benefit from a single model representing the experience of many companies. Regulators are concerned with establishing fair pricing of insurance policies and ensuring that insurers have sufficient assets to meet contractual obligations. A single model can help regulators examine loss experiences of several companies, using covariate information to comparably account for the risks underwritten by these companies. Moreover, regulators can use these comparisons for detecting fraud and further inspecting unusually high or low frequency of losses (that may be suspect as indicated by the risk rating factors as covariates).

Reinsurers are the 'insurers of insurance companies' and they take on layers of risks so that insurers are able to diversify their loss exposure. Naturally, reinsurers are interested in the loss distributions that they are accepting. Predictions at the company level are important to prices charged by reinsurers.

In the United States, the Society of Actuaries ('SOA') collects intercompany data through experience studies. As noted in Iverson et al. (2007), "one of the key elements that led to the creation of the Actuarial Society of America in 1889 was the need for an independent body to collect and report upon experience." The SOA publishes descriptive statistics based on the data collected from participating insurers and these "intercompany reports of experience are considered a proxy for the state of the industry with companies using these results to benchmark their own experience" (Iverson et al. (2007)). This practice of collecting intercompany data extends to several parts of the globe including Australia through its main professional body, the Institute of Actuaries of Australia. In Europe, for instance, the Dutch Center for Insurance Statistics ('Centrum voor Verzekeringsstatistiek') collects intercompany data from various insurance branches and publishes summary statistics on a regular basis.

Despite several parties (insurers, reinsurers, regulators and actuarial organizations like the SOA) interested in the analysis of intercompany data on claim statistics, sound statistical research in this area is still lacking.

The primary contributions of this research paper are therefore threefold. Firstly, we develop the connection between hierarchical credibility and multilevel statistics, a discipline that is generally unknown in actuarial science. We go beyond the two level structures often found in panel data. Credibility is a classical actuarial approach for experience rating (and Hickman and Heacox (1999) claimed it to be one of the cornerstones of actuarial mathematics). Secondly, with the growing popularity of the generalized count distributions in actuarial science, we extend their applications towards more than two level data sets. Bayesian statistics and MCMC sampling are used for estimation in our model specifications. Deb et al. (2006) is an example of a Bayesian analysis in the econometric literature that is related to this article. Thirdly, we provide modeling and a detailed analysis of intercompany data on fleets, which, as alluded earlier, has been rather scarce in the actuarial literature. We include goodness–of–fit statistics for the various models and illustrate their predictive capability.

The paper has been structured as follows. Section 2 gives background on the data used in the analysis. Model specification, results and prediction for claim counts is covered in Section 3, 4 and 5. Section 6 concludes.

2 Intercompany Insurance Claims Data

2.1 Background

We investigate a dataset with policy exposure information, covariates and claim counts for vehicle insurance portfolios of general insurance companies in Singapore. With 'exposure' defined as the fraction of a year during which the policyholder pays for insurance. The source of this intercompany dataset is the General Insurance Association ('GIA') of Singapore (see the organization's webpage http://www.gia.org.sg), an organization with membership consisting of general insurers in Singapore. In Singapore, just as in many parts of the world, motor insurance is compulsory and it is not surprising to find it to be one of the most important general insurance lines of business.

Two files were examined: the policy and the claims files. The policy file consists of records of policyholders with vehicle insurance coverage purchased from a general insurer during the period 1993-2001. Each vehicle is identified with a unique code. For fleet policies, no information on the driver of the vehicle is available, since a vehicle may be used by several drivers. Thus, the unit of observation in our analysis is a registered vehicle insured, broken down according to their exposure in each calendar year 1993 to 2001. The claims file provides a record of each accident claim that has been filed with the insurer during the observation period and is linked to the policyholder file.

All policies in the sample have a comprehensive coverage that includes coverage for third party injury and property damage as well as damage to one's own vehicle. Each vehicle is followed over a period of (maximum) nine years: January 1993 until December 2001. However, vehicles, as well as fleets, enter and exit the sample. Vehicles with questionable information on some variables such as the vehicle capacity or the year the car was manufactured were removed from the sample and therefore ignored in the analysis.

The hierarchical structure of the data lends itself naturally to multilevel modeling with four different levels for this data set. At the highest level, we analyze ten insurance companies (using c to denote a company). To keep the amount of analysis and exposition manageable, ten companies, labeled 1 to 10, were randomly drawn from 27 companies available in the GIA's entire database. At the next level, from these 10 companies, we consider 6,763 fleets (f). Level two consists of 16,437 vehicles (v) that we observe over time (t), for a total of 39,120 level one observations.

2.2 Data Characteristics

The empirical distribution of the observed claim counts is in Table 1. The table (2^{nd} column) illustrates that about 88% percent of the observations are zeros. At most 5 claims during one time period have been observed. Table 1 also describes the claim distribution by company. The number of observations and exposures show that the companies are roughly the same size, with no one company dominating the market. Table 1 suggests substantial differences among companies; the mean claim count for company 3 is quite small compared to companies 2, 9 and 10.

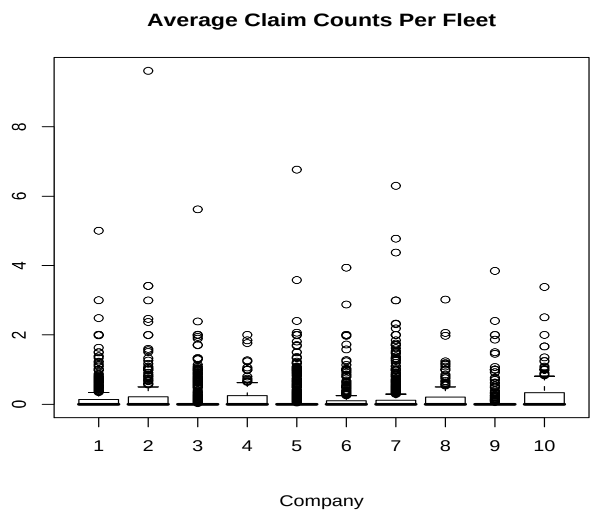
Table 1: *Claims by company*

		Percentage of Claims by Company									
Count	All	1	2	3	4	5	6	7	8	9	10
0	87.82	88.27	81.68	94.68	87.71	89.43	88.83	87.44	86.86	88.78	87.28
1	10.49	10.23	15.11	4.96	10.55	9.30	9.74	11.09	11.13	9.57	10.85
2	1.41	1.30	2.73	0.30	1.43	0.96	1.10	1.26	1.62	1.37	1.71
3	0.22	0.18	0.36	0.06	0.29	0.19	0.20	0.19	0.34	0.24	0.17
4	0.04	0.03	0.12	0.00	0.00	0.06	0.10	0.02	0.05	0.04	0.00
5	0.01	0.00	0.00	0.00	0.02	0.06	0.04	0.00	0.00	0.00	0.00
Claims	5,557	528	1,096	191	603	398	669	891	318	328	535
Observations	39,120	3,920	4,951	3,327	4,191	3,225	5,105	6,251	2,040	2,487	3,623
Exposure	30,560	3,106	4,440	2,480	3,240	2,497	3,978	5,023	1,635	1,505	2,656
Mean Claim	0.14	0.17	0.25	0.08	0.19	0.16	0.17	0.18	0.19	0.22	0.20
Fleets	6,763	841	270	1,229	270	1,279	646	1,286	335	268	339

Notes: 'Exposure' is the total exposure time per company (in policy years).
'Mean' is the sum of claim counts divided by the total exposure.

Figure 1 shows the distribution of claim counts at the fleet level. Specifically, for each fleet, the average claim count (per unit of exposure) was computed and the distribution of these averages appears in Figure 1, by company. One can observe company effects, as in Table 1, in the sense that the average number of claims reported by company 3 is very low, whereas the averages from companies 2 and 10 are rather high. Company 9 is special in that it has the lowest exposure and yet one of the largest claims per unit of exposure. However, at fleet level (see Figure 1), 81% of the fleets in this company reported zero claims in total (compare this with e.g. company 10 where only 51% of the fleets stayed claim–free during the observation period).

Figure 1: *Boxplots of a fleet's average number of claims, by company. This figure shows that the distribution of claim behavior differs by company.*



As we can see from Table 1, our sample is unbalanced. Vehicles are bought and sold by fleets periodically; fleets themselves merge, go out of business and start up new. For our data, most of the imbalance is due to fleet movement; the mean time that the fleet was in the data was only 1.4 years. To capture this movement, we examine the growth in the size of the fleet ('FleetChange', see Table 3) as well as the length of the time period that a vehicle stays within the same fleet ('TLInFleet', see Table 2).

Measurable characteristics at the level of the vehicle are summarized in Table 2. Because the data are from an intercompany study, no specific information was available to identify the fleet nor the insuring company (such as the branch where the fleet is operating, details on the financial structure of the company, et cetera). To see how this information might be useful, Angers et al. (2006) uses the sector of activity of the carrier as an explanatory variable in their regression analysis. To compensate for this deficiency, averages at the level of the fleet and

Table 2: *Vehicle level explanatory variables*

Categorical Covariates	Description	Percentage		
Vehicle Type	Car	54%		
	Motor	41%		
	Truck	5%		
Private Use	Vehicle is used for private purposes	31%		
	Vehicle is used for other than private purposes	69%		
NCD	'No Claims Discount' at entry in fleet: based on previous accident record of policyholder. The higher the discount, the better the prior accident record.			
	NCD $= 0$	83%		
	NCD > 0	17%		
Continuous Covariates		**Minimum**	**Mean**	**Maximum**
Vehicle Age	The age of the vehicle in years, at entry in fleet	0	4.22	33
Cubic Capacity	Vehicle capacity for cars and motors	124	1,615	6,750
Tonnage	Vehicle capacity for trucks	1	7.6	61
TLengthEntry	Time (in years) vehicle was in the sample, before entering the fleet	0	0.35	6.75
TLInFleet	Time (in years) vehicle was in same fleet prior to the current observation period	0	0.13	5
TLength	(Exposure) Fraction of calendar year for which insurance coverage is purchased	0.006	0.78	1

Table 3: *Fleet and company level explanatory variables*

Covariate	Description	Minimum	Mean	Maximum
Fleet Level				
AvNCD	Average of No Claims Discount at entry in the fleet	0	6.3	50
AvTLengthEntry	Average of TLengthEntry	0	0.59	6.75
AvTLength	Average of cumulative time period spent in fleet	0	1	3.64
AvVAge	Average of vehicle age at entry in the fleet	0	4.75	27.33
AvPrem	Average of premium paid (per unit of exposure)	0.01	1.3	59.56
FleetCap	Number of vehicles in the fleet	1	4.56	1,092
FleetChange	Size of fleet in current year divided by size in previous year (when observed in both periods)	0.015	3.1	31
Company Level				
NumFleets	Number of fleets in the company	268	942	1,286
NumVeh	Number of vehicles in the company	1,319	3,084	5,394
NumCars,	Number of cars, trucks and motorcycles	391	1,652	4,453
NumTrucks,	in the company	224	1,259	3,019
NumMotors		0	170	888

company are created which are listed in Table 3. The averages in the upper part of the table are computed at fleet level, e.g. 'AvPrem' is the total premium paid by all vehicles in the fleet, divided by the total period (in years) for which insurance is guaranteed by the fleet. At the company level, for informative purposes, we give descriptive statistics of number of fleets and vehicles, together with each type of vehicle (cars, trucks, and motorcycles). Neither of these were explicitly considered as explanatory variables because the mean parameters in the count models later considered already account for the level of exposure as explained in Section 3.2.

Categorized versions of the covariates are used in our multilevel specifications (see Appendix Tables 11 and 12). Using categorizations is consistent with insurance company practice and with the literature on non-life insurance (see e.g. Desjardins et al. (2001), Angers et al. (2006) and Denuit et al. (2007)).

3 Multilevel Count Models

We use multilevel modeling for this four–level data set (vehicles followed over time, grouped in a fleet policy issued by a company). Multilevel models allow us to incorporate the hierarchical structure of the data by specifying random effects at the vehicle, fleet and company levels. These random effects represent unobservable characteristics at each level. At the vehicle level, the missions assigned to a vehicle or unobserved driver behavior may influence the riskiness of a vehicle. At the fleet level, guidelines on driving hours, mechanical check–ups, loading instructions and so on, may influence the number of accidents reported. As described in Section 1, at the insurance company level, underwriting and claim settlement practices may affect claims.

We employ the standard nested structure of multilevel models. As described in Section 2.2, there is considerable movement of fleets and vehicles in and out of the sample. We capture this in part through explanatory variables such as the growth of the fleet ('Fleetchange') and the time that vehicles have been in the same fleet ('TLInFleet'). We follow industry practice and assume that the residual reasons for movement are not related to the response variable. In principle, our assumptions of nested random effects may be violated when fleets switch insurance companies or when vehicles switch fleets. For the former, below we examine alternative models that use fixed insurance company effects (as a robustness check). For the latter, when a vehicle switches fleets, we assume that (1) differences in management practices between the two fleets as well as (2) vehicle inspection by new fleet owners dominate any unobserved (unaccounted for) vehicle effects. Thus, the vehicle is treated as a new entrant in the fleet and prior claims are not taken into account when estimating the random effect of the current fleet.

3.1 Count Models

We will not only investigate Poisson regression, but also negative binomial, zero–inflated Poisson and hurdle Poisson models. Both the Poisson and the negative binomial are commonly used for count data. The zero-inflated Poisson distribution with parameters p and λ, ZIP(p,λ), is given by

$$\mathrm{Pr}_{\mathrm{ZIP}}(Y=y|p,\lambda)=\begin{cases} p+(1-p)\mathrm{Pr}_{\mathrm{Poi}}(Y=0|\lambda) & y=0 \\ (1-p)\mathrm{Pr}_{\mathrm{Poi}}(Y=y|\lambda) & y>0 \end{cases}. \tag{1}$$

Here, the term $\mathrm{Pr}_{\mathrm{Poi}}(Y=y|\lambda)$ denotes a Poisson distribution with mean λ. The zero-inflated model provides a mixture of a point mass at zero together with a Poisson random variable. This representation is well–suited to handle the "excess" number of zeros reported in Table 1, where "excess" is relative to a known distribution. For example, with the Poisson distribution a single parameter determines both the mean and the probability of zero claims. For many empirical count distributions, the frequency of zeros exceeds that suggested by the mean number of claims. Another way of handling the many zeros is through the "hurdle" Poisson distribution with parameters p and λ, herewith denoted as HUR(p, λ), given by

$$\mathrm{Pr}_{\mathrm{Hur}}(Y=y|p,\lambda)=\begin{cases} p & y=0 \\ \frac{1-p}{1-\mathrm{Pr}_{\mathrm{Poi}}(0|\lambda)}\mathrm{Pr}_{\mathrm{Poi}}(Y=y|\lambda) & y>0 \end{cases}. \tag{2}$$

Hurdle models are generally motivated with sequential decision–making processes. In the case of insurance claims, the decision to leave the zero state and report a claim involves one probability model (with probability $1-p$) whereas subsequent claims follow another model (a truncated Poisson in equation (2)).

In the absence of covariate information, these count distribution models are each fitted to the 'rough' dataset, without covariates. A comparison of their performance is illustrated in Table 4, using the Pearson chi-square (χ^2) statistic and deviance information criterion (DIC, see Spiegelhalter et al. (2002)) to summarize the fit. WinBUGS is used for the Bayesian estimation of all models reported in this paper. Table 4 suggests that the negative binomial is the best candidate model.

Section 3.2 is a summary of the multilevel models that we investigated for our data. The corresponding results are presented and discussed in Section 4.

3.2 Model Specifications

3.2.1 Hierarchical Poisson Models

The starting point is a hierarchical Poisson model with random intercepts for the vehicle, fleet and company. Specifically, we begin with a Bayesian implementation of Jewell's hierarchical

Table 4: *Observed and expected claim counts for the various count distributions*

Number of Claims	Observed Frequency	Poisson	Negative Binomial	ZIP	Hurdle Poisson
0	34,357	33,940	34,362	34,357	34,357
1	4,104	4,821	4,079	4,048	4,048
2	551	342	577	641	641
3	86	16	86	68	68
4	17	1	13	5	5
≥ 5	5	0.016	2	0.34	0.34
Pearson χ^2		566	7.1	70.15	70.15
DIC		34,034	31,020	33,586	33,586

Note: No covariates are included in these model specifications

model for counts. Jewell's scheme (see Kaas et al. (2008) and Antonio and Beirlant (2007)) is the traditional actuarial approach for experience rating with hierarchical data structure. It is distribution–free in its original specification, but can be interpreted as a random effects model under normality assumptions (see Frees et al. (1999)). Our specification is

$$Y_{c,f,v,t} \sim \text{Poi}(\lambda_{c,f,v,t}) \quad \text{with} \quad \lambda_{c,f,v,t} = e_{c,f,v,t} \exp\left(\gamma + \epsilon_c + \epsilon_{c,f} + \epsilon_{c,f,v}\right). \tag{3}$$

Here, $Y_{c,f,v,t}$ denotes the claims observed in year t for vehicle v, which is insured under fleet f in company c. The term $e_{c,f,v,t}$ is an exposure variable that gives the length of time during calendar year t for which the vehicle has insurance coverage. Further, γ is the intercept, ϵ_c is a random company effect, $\epsilon_{c,f}$ is a random effect for the fleet within the company and $\epsilon_{c,f,v}$ is a random effect for the vehicle within the fleet.

The hierarchical Poisson model extends Jewell's scheme by incorporating risk factors in terms of explanatory variables. We consider

$$\begin{aligned} Y_{c,f,v,t} \sim \text{Poi}(\lambda_{c,f,v,t}) \quad &\text{with} \quad \lambda_{c,f,v,t} = e_{c,f,v,t} \exp\left(\eta_{c,f,v,t} + \epsilon_c + \epsilon_{c,f} + \epsilon_{c,f,v}\right) \\ &\text{and} \quad \eta_{c,f,v,t} := \gamma + \boldsymbol{X}_c\boldsymbol{\beta}_4 + \boldsymbol{X}_{c,f}\boldsymbol{\beta}_3 + \boldsymbol{X}_{c,f,v}\boldsymbol{\beta}_2 + \boldsymbol{X}_{c,f,v,t}\boldsymbol{\beta}_1. \end{aligned} \tag{4}$$

The explanatory variables used in the systematic component $\eta_{c,f,v,t}$ are:

- $\boldsymbol{X}_{c,f,v,t}$: TLInFleet, FleetChange;
- $\boldsymbol{X}_{c,f,v}$: VehicleType, Capacity Cubic, Tonnage, VAgeEntry, TLengthEntry, Private;
- $\boldsymbol{X}_{c,f}$: AvPrem, AvTLength, AvTLengthEntry, AvNCD; and

- $\boldsymbol{X}_c$: none.

However, for fleets with only one vehicle (there are 6,245 of such fleets in the sample) the general specification of the linear predictor is slightly modified. With only one vehicle per fleet, the vehicle and fleet level coincide. Therefore, no fleet random effect is included for such fleets, neither are averages at the level of the fleet used. Apart from the model in (4), we also consider a Poisson hierarchical model with a random effect for the company and fleet but without a random effect for the vehicle.

Our distributional assumptions for the random effects in the Poisson hierarchical models (as in (4)) slightly differ from those traditionally used by Jewell and in multilevel modelling:

$$\epsilon_c \sim N(-\frac{\sigma_c^2}{2}, \sigma_c^2), \ \epsilon_{c,f} \sim N(-\frac{\sigma_{c,f}^2}{2}, \sigma_{c,f}^2), \text{ and } \epsilon_{c,f,v} \sim N(-\frac{\sigma_{c,f,v}^2}{2}, \sigma_{c,f,v}^2). \tag{5}$$

This specification is now common in the actuarial literature because means are unchanged with the introduction of random effects. For example, basic calculations show that $\mathrm{E}[\exp(\epsilon_c)] = 1$, and similarly for $\epsilon_{c,f}$ and $\epsilon_{c,f,v}$.

Now, using the specifications in (4) and (5), the *a priori* mean, $\mathrm{E}[Y_{c,f,v,t}]$, is given by

$$\mathrm{E}[Y_{c,f,v,t}] := \lambda_{c,f,v,t}^{\text{prior}} = e_{c,f,v,t} \exp{(\eta_{c,f,v,t})}. \tag{6}$$

The *a posteriori* premium, $\mathrm{E}[Y_{c,f,v,t}|\epsilon_c, \epsilon_{c,f}, \epsilon_{c,f,v}]$, then becomes

$$\mathrm{E}[Y_{c,f,v,t}|\epsilon_c, \epsilon_{c,f}, \epsilon_{c,f,v}] \quad = \quad \lambda_{c,f,v,t}^{\text{prior}} \exp(\epsilon_c + \epsilon_{c,f} + \epsilon_{c,f,v}). \tag{7}$$

In our Bayesian analysis, the posterior distributions of (6) and (7) are used for ratemaking. Examples follow in Section 5. Specification (7) explicitly shows how *a posteriori* corrections are made to the *a priori* premium.

The prior distributions used in the Bayesian analysis are selected as follows (similar specifications are used for the other models discussed in this Section):

(i) for the regression coefficients in $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4$: a normal prior with a variance of 10^6;

(ii) for the inverse of the variance components: gamma priors $\Gamma(0.001, 0.001)$.

We also investigate a "fixed company effects" version of the hierarchical Poisson model. In this version, the company effects ϵ_c are (fixed) unknown parameters to be estimated. For our data, there are only 9 parameters (for 10 companies, including an intercept term) that need be estimated.

3.2.2 Hierarchical Negative Binomial Model

Because the negative binomial provides a good fit to the 'raw' count data in Table 4, a negative binomial multilevel regression model is considered as well. This regression model uses the same covariate information as in Section 3.2.1. Thus, we assume that $Y_{c,f,v,t}$ has a negative binomial distribution with scale parameter τ and location parameter

$$\mu_{c,f,v,t} = e_{c,f,v,t} \exp\left(\eta_{c,f,v,t} + \epsilon_c + \epsilon_{c,f}\right), \tag{8}$$

where random effects for the company and fleet are used. For $\eta_{c,f,v,t}$ the same set of covariates is used as in the hierarchical Poisson model (given right below equation (4)). The prior distribution for τ was $\tau = \exp(\tau^{\star})$ with $\tau^{\star} \sim N(0, 10^6)$. From a predictive point of view, we illustrate in Section 5 that models without a random vehicle effect (as in (8)) rely on the history of the whole fleet to which the vehicle belongs (and on the history of the company), but do not use the history of a vehicle separately. This is in contrast with (4).

3.2.3 Hierarchical Zero-Inflated Poisson Models

Two types of zero–inflated Poisson models were investigated. For the first specification, we have:

$$\begin{aligned} Y_{c,f,v,t} &\sim \text{ZIP}(p, \lambda_{c,f,v,t}) \\ \text{where } \lambda_{c,f,v,t} &= e_{c,f,v,t} \exp\left(\eta_{c,f,v,t} + \epsilon_c + \epsilon_{c,f}\right). \end{aligned} \tag{9}$$

The specification of $\eta_{c,f,v,t}$ is the same as in the previous models. Prior specifications are similar as before, completed with $p \sim \text{Beta}(1, 1)$ as the prior for the additional proportion of zeros.

In a second hierarchical ZIP model, we let the extra proportion of zeros be fleet-specific and use $p_{c,f} \sim \text{Beta}(1, b)$. The prior for b is $\log(b) \sim N(0, 10^6)$. Thus,

$$\begin{aligned} Y_{c,f,v,t} &\sim \text{ZIP}(p_{c,f}, \lambda_{c,f,v,t}) \\ \text{where } \lambda_{c,f,v,t} &= e_{c,f,v,t} \exp\left(\eta_{c,f,v,t} + \epsilon_c + \epsilon_{c,f}\right), \end{aligned} \tag{10}$$

(with $\eta_{c,f,v,t}$ as before). We illustrate in Section 5 that predictive premiums obtained with (10) do not only depend on the number of past claims, but also on the claim–free period of a fleet (through $p_{c,f}$).

3.2.4 Hierarchical Hurdle Poisson Model

For the hurdle Poisson model with level specific explanatory variables and random effects, the following specification is used:

$$\begin{aligned} Y_{c,f,v,t} &\sim \text{HUR}(p_{c,f,v,t}, \lambda_{c,f,v,t}) \\ \text{where logit}(p_{c,f,v,t}) &= \eta_{c,f,v,t,\text{Bin}} + \epsilon_{c,\text{Bin}} + \epsilon_{c,f,\text{Bin}}, \\ \text{and } \lambda_{c,f,v,t} &= e_{c,f,v,t} \exp\left(\eta_{c,f,v,t} + \epsilon_c + \epsilon_{c,f}\right). \end{aligned} \tag{11}$$

The risk factors in the systematic component $\eta_{c,f,v,t,\text{Bin}}$ are

- intercept, TLInFleet, FleetChange;
- VehicleType, Private, VehicleAge, Capacity Cubic, Tonnage;
- AvTLength, AvTLengthEntry, AvPrem, AvNCD.

For the other systematic component, $\eta_{c,f,v,t}$, the explanatory variables VehicleType, Capacity Cubic, Tonnage, Private and VehicleAge are included, as well as random fleet and company effects. Note that this second part of the model (over the hurdle) is fitted to only 12% of the original data set. Distribution specifications for the random effects are those in (5). The random effects in the zero and non-zero part are independent of each other. These specifications however do not lead to an explicit expression for the *a priori* mean. We will illustrate in Section 5 that, as model (10) does, the hurdle model takes the claim–free period of a fleet, as well as the number of past claims, into account when future premiums are set.

4 Model Estimation Results

4.1 Hierarchical Poisson Models

Estimated claim frequencies are in Table 5. In the table we compare the results obtained with Jewell's hierarchical model (full version), Jewell's hierarchical model with just fleet and company specific intercepts (no covariates), a Poisson regression without random effects, the Poisson regression model with random intercepts for vehicles, fleets and companies, and the Poisson regression model with random vehicle and fleet effects, but fixed company effects. The table reports estimated claim counts obtained from hierarchical Poisson analysis, as well as the Pearson χ^2 statistic and DIC. For every count model, two parallel chains were run; 15,000 iterations each with burn-in of 500 observations. We conclude that the Poisson multilevel model in (4), as well as the fixed effects version, outperform the other specifications. The parameter estimates from a Poisson model with fixed company effects (not shown) are very similar to those obtained with model (4), and so are the estimated claim counts in Table 5. Because of the proximity of these two models, we henceforth restrict considerations to the random effects version of the multilevel Poisson model in (4).

Table 5: *Estimated claim counts obtained from Bayesian Poisson hierarchical analyses*

Number of Claims	Observed Frequency	Poisson Regression No RE	Poisson Jewell No Vehicle RE	Poisson Jewell (3)	Poisson Multilevel (4)	Poisson Multilevel Fixed Effects
0	34,357	34,030	34,180	34,300	34,316	34,318
1	4,104	4,647	4,395	4,204	4,170	4,167
2	551	410	490	529	538	538
3	86	28	52	76	79	79
4	17	2	6	13	14	14
≥ 5	5	0.19	0.69	2	3	3
Pearson χ^2		468.2	97.1	10.4	4	3.9
DIC		32,480	31,331	31,040	30,642	30,638

Notes: 'RE' stands for random effects. The Poisson regression and multilevel models include covariates; the 'Jewell' models do not.

4.2 Hierarchical Negative Binomial, Zero-Inflated and Hurdle Poisson Models

Table 6 compares estimated claim amounts for the preferred hierarchical Poisson model (4) and alternatives described in Sections 3.2.1- 3.2.4. This table indicates that the Poisson and negative binomial outperform the other models. We also remark that, among the negative binomial, zero-inflated Poisson and hurdle Poisson, the hurdle Poisson regression models have the advantage of allowing for the fastest MCMC sampling. In Section 5 we compare these models based on how they use information from the past to construct future premiums.

For the case of the hierarchical negative binomial model from Section 3.2.2 the 95% interval estimates of the parameters are visually displayed in Figure 2. In addition, for the same model, Figure 5 provides a graphical display of the mixing and convergence of the chains used to produce the interval estimates. Figure 5 also displays the resulting numerical values of these interval estimates. Similar interval estimates of the parameters for the other models have been produced. We do not include them in this paper because interpretations of the parameter estimates would be indeed quite similar. However, there may have been slight differences in the selection of explanatory variables for different model specifications.

Table 6: *Estimated claim counts obtained from Bayesian hierarchical analyses*

Number of Claims	Observed Frequency	Poisson (4)	Negative Binomial	ZIP (9)	Hurdle Poisson (11)
0	34,357	34,316	34,357	34,336	34,360
		(34,200;34,430)	(34,240;34,480)	(34,210;34,460)	(34,240;34,480)
1	4,104	4,170	4,100	4,123	4,140
		(4,073;4,266)	(3,991;4,209)	(4,013;4,238)	(4,030;4,252)
2	551	538	556	571	536
		(514;562)	(529,584)	(539;605)	(504;570)
3	86	79	86	76	72
		(72;87)	(76;97)	(67;85)	(64;82)
4	17	14	16	11	10
		(11;16)	(12;19)	(9;13)	(8;13)
≥ 5	5	3	3	2	1.5
		(2;4)	(2;4.4)	(1;2)	(0.9;2.3)
Pearson χ^2		4	1.4	9.9	16.6
DIC		30,642	30,961	30,982	31,645

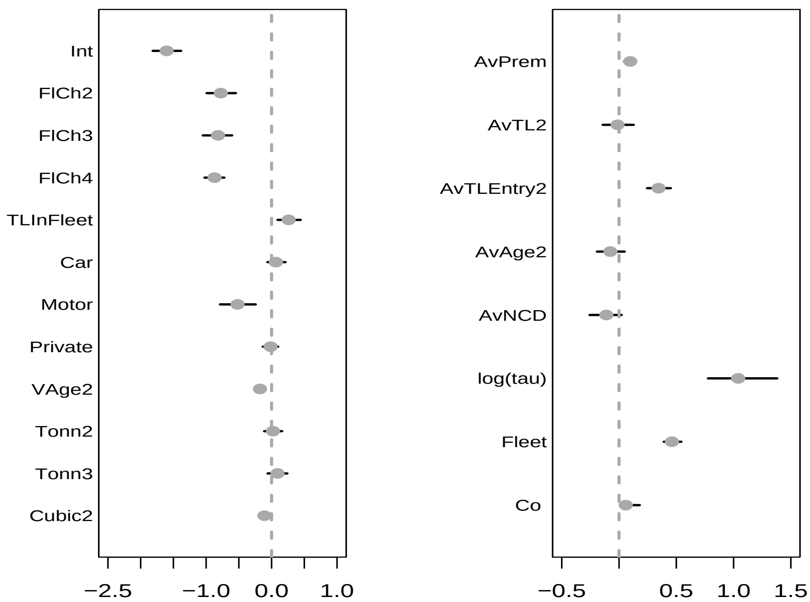
Note: 95% interval estimates are given in parens

The selection of the explanatory variables (or covariates) was partly motivated by what was available in our dataset. See Tables 2 and 3 for a description of these variables. Insurers are typically restricted with the type of information they can retain in their databases. In addition, because only fleet policies are considered in this paper, it adds the complexity of recording information regarding driver characteristics, because several drivers may use a single vehicle. Therefore, at vehicle level, none of our covariates contain driver characteristics. Studying the movements of vehicles in preliminary data analysis inspired us to investigate the effect of the time varying covariates 'FleetChange' and 'TLInFleet' (see Section 2.2).

At the fleet level, we examined the effects of premium ('AvPrem'), the level of NCD ('AvNCD'), the average time period during which a vehicle has been insured before entering the fleet ('AvTLengthEntry'), the time it stays on average in the same fleet ('AvTLength'), and the changes in the size of the fleet from period to period ('FleetChange'). According to Figure 2, the average level of premium paid by the fleet policy has a significant positive effect on the average claim counts; not surprisingly, since it is usually actuarially fair to say that the level of premium is directly related to the level of claims. Changes in the level of premium could motivate fleet owners to better manage its vehicles by either discarding problematic vehicles or maintaining them to avoid further claims. As shown in Table 12, the variable 'FleetChange' has four categories with the "Not Defined" category as the level of reference. Because 'FleetChange' refers to the ratio of the size of the fleet in the current year relative to the previous year, the fleet has to be observed in both periods for this to be defined. This led us to the indicator variables in Figure 2 ('FLCh2', 'FlCh3', and 'FlCh4') with each of them respectively referring to a reduction, no change, or an increase in the fleet size. Figure 2 shows that our model estimates provide an indication of the significance of these changes in the fleet size from period to period: each change in fleet size has a negative effect. When a vehicle in year t is in the same fleet as it was in year $t-1$, the variable 'TLInFleet' is strictly positive (and the corresponding indicator in Table 11 takes the value 1). At the same time, one of the 'FleetChange' indicator variables equals one, since the fleet necessarily is observed in the current and past year. In Figure 2 we see that their combined effect is always negative and significant. This implies that staying in the same fleet reduces the expected number of claims.

At the vehicle level, few risk factors have a statistically significant effect on the average number of claims. This appears to be in agreement with the findings by Frees and Valdez (2008) who even investigated datasets that involved non-fleet policies where more driver and vehicle characteristics are more readily available. In our model estimates, a few of the risk factors that intuitively makes sense provided an impact on the average number of claims included. For instance, motorcycles ('motor') report significantly fewer claims than the reference class (i.e. trucks) although cars ('car') does not appear to have significantly different number of claims to trucks, and heavy trucks ('Tonnage' > 8) report more claims.

Figure 2: *95% interval estimates. Each interval estimate corresponds to a parameter of the negative binomial model* (8). *The dashed line emphasizes whether the interval includes zero.*



5 *A Priori* Premiums and *A Posteriori* Updates

Two applications of the hierarchical models are now considered. Section 5.1 establishes differences among insurance companies and among fleets by showing the posterior distribution for each level. Section 5.2 uses these distributions to update insurance premiums, an important exercise for insurance companies and regulators who monitor their behavior.

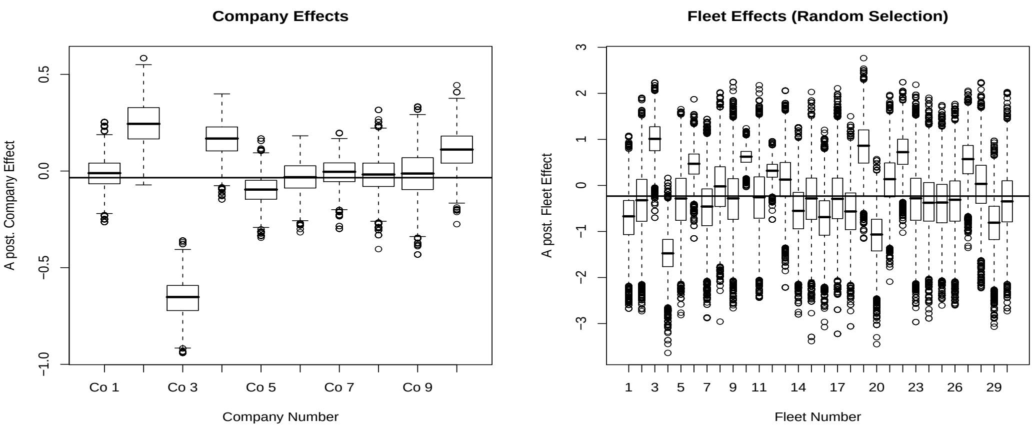
5.1 Posterior distributions for the random effects

With the hierarchical models, we can also "summarize" the random effects. We focus on the hierarchical negative binomial model (see (8)). Figure 3 illustrates the posterior distributions for the company effects as well as for a random selection of fleet effects.

Are there any important insurance company effects? The left-hand panel of Figure 3 helps the analyst respond to this question through a summary of the distribution of each company effect. This panel shows, even after controlling for covariates and fleet level effects, that company 3 is lower than competitors, especially compared to companies 2 and 10. Unlike the raw counts in Table 1, company 4 appears to have a high effect whereas company 9 is more typical of other companies.

The right-hand panel of Figure 3 shows fleet effects for a random selection of fleets. This panel shows the heterogeneity in the distribution of fleet effects. Note also the different scales in the two panels; the vertical scale on the company effects is much more narrow than the fleet effects. This indicates that fleets effects are more variable than company effects.

Figure 3: *Illustration of a posteriori distributions of company effects and a random selection of fleet effects for the negative binomial model. A horizontal line is plotted at the mean of the random effects distribution (as in* (5)*).*



5.2 *A posteriori* updates to *a priori* premiums

The purpose of the data analysis is a sound statistical approach to premium rating for intercompany data. How should a reinsurer or regulator translate the company effects that became apparent in the descriptive Table 1 into an accurate prediction for the expected number of claims? The posterior distribution of the random company effects is used for this purpose.

The different distributions used in Section 3.2 represent different styles of penalizing for past claims. For instance, the zero-inflated model with fleet–specific $p_{c,f}$ (see (10)) and the hurdle Poisson model in (11) not only use the number of past claims, but also the claim–free period of a fleet. The various distribution models considered in this paper give the user the choice of which style is suitably adoptable to his philosophy.

We illustrate how the reported claims history *a posteriori* updates the *a priori* premium. In Section 5.2.1 we follow three vehicles and illustrate how the various models from Section 3 update the *a priori* premiums. Section 5.2.2 summarizes *a posteriori* updates for a specific model and all fleets. Recall from Section 3 that $\mathrm{E}[Y_{c,f,v,t}]$ is used for the *a priori* premium and $\mathrm{E}[Y_{c,f,v,t}|\epsilon_c, \epsilon_{c,f}, \epsilon_{c,f,v}]$ is used for the *a posteriori* premium. For the reader's convenience, Table 7 is a summary of the expressions for both premiums and resulting bonus–malus factors for the model specifications investigated in this paper (when explicit expressions exist). Hereby the bonus–malus factor ('BMF') is the ratio (*a posteriori* premium/*a priori* premium). These BMFs are standards used in the insurance industry for penalizing or rewarding customers according to their historical claims experience. A BMF > 1 indicates some penalty required, while a BMF < 1 indicates the opposite. See Lemaire (1995) for more details. Since we rely on Bayesian statistics for our estimations, the illustrations below use the posterior mean of the expressions displayed in the BMF column of Table 7.

Table 7: *A priori and a posteriori premiums, with Bonus Malus Factors, for several distributions*

Distribution	*A Priori* Premium $\mathrm{E}[Y_{c,f,v,t}]$	*A Posteriori* Premium $\mathrm{E}[Y_{c,f,v,t}\vert\epsilon_c, \epsilon_{c,f}, \epsilon_{c,f,v}]$	Bonus Malus Factor BMF
Poisson	$\lambda^{\text{prior}}_{c,f,v,t}$ $= e_{c,f,v,t}\exp(\eta_{c,f,v,t})$	$\lambda^{\text{prior}}_{c,f,v,t}\exp(\epsilon_c+\epsilon_{c,f}+\epsilon_{c,f,v})$	$\exp(\epsilon_c+\epsilon_{c,f}+\epsilon_{c,f,v})$
Negative Binomial	$\mu^{\text{prior}}_{c,f,v,t}$ $= e_{c,f,v,t}\exp(\eta_{c,f,v,t})$	$\mu^{\text{prior}}_{c,f,v,t}\exp(\epsilon_c+\epsilon_{c,f})$	$\exp(\epsilon_c+\epsilon_{c,f})$
ZIP - type 1	$(1-p)\lambda^{\text{prior}}_{c,f,v,t}$, where $\lambda^{\text{prior}}_{c,f,v,t} = e_{c,f,v,t}\exp(\eta_{c,f,v,t})$	$(1-p)\lambda^{\text{prior}}_{c,f,v,t}\exp(\epsilon_c+\epsilon_{c,f})$	$\exp(\epsilon_c+\epsilon_{c,f})$
ZIP - type 2	$(1-\frac{1}{1+b})\lambda^{\text{prior}}_{c,f,v,t}$	$(1-p_{c,f})\lambda^{\text{prior}}_{c,f,v,t}\exp(\epsilon_c+\epsilon_{c,f})$	$\frac{1-p_{c,f}}{1-1/(1+b)}\exp(\epsilon_c+\epsilon_{c,f})$
Hurdle	No explicit expressions		

5.2.1 Numerical illustrations

Let us start from the model in (4). To update the *a priori* premium, this model uses the history of the vehicle, the history of the fleet to which it belongs and the history of the company. In Tables 8 and 9 we follow three vehicles to illustrate the mechanism of experience rating with each of the model specifications investigated in this paper. In the first illustration from Table 8, for fleet number 2,814, the BMFs for all vehicles are above 1, but the BMF for the vehicle that reports 1 claim is much higher (2.05) than the BMF for the claim-free vehicles (1.56 and 1.58). Checking the corresponding results for the hierarchical negative binomial model and the ZIP with fixed p, the BMF for all vehicles is > 1 and in between those reported in the first illustration in Table 8. The latter models calculate BMFs at the fleet level, a natural point in the hierarchy because it is at this level where an insurance contract between a fleet and insurance company is written. Hence, fleet level BMFs can be used for premium renewals. The first illustration in Table 8 shows BMFs calculated at the vehicle level. This information could also be used for contracts written at the fleet level; as the fleet composition changes through the retirement or sale of vehicles, the total fleet premium should reflect the changing composition of vehicles. Vehicle level BMFs will allow prices to depend on the vehicle composition of fleets. We anticipate that pricing actuaries will find both set of findings useful.

Comparing the results in Tables 8 and 9 we see that *a priori* premiums obtained with the different model specifications closely correspond. The zero-inflated model with fleet-specific $p_{c,f}$ (see (10)) and the hurdle poisson model in (11) take the claim-free period of a fleet into account. For panel data this feature was made explicit in Boucher et al. (2009) and Boucher et al. (2008). Compare the results for fleet 2,814 between the various specifications: in the Poisson, NB and ZIP with p fixed, the BMF for this fleet is 1.76/1.71/1.8. In ZIP model (10) this drops to 1.37 and in the hurdle model even to 1. That is because these last two model specifications not only use the number of registered claims, but also the claim-free periods (which is here 17 out of a total of 18.5 years).

In Table 10 some artificial scenarios are investigated. Fleet 4,672 originally belongs to company 3; this fleet reported 5 claims over a period of 20.4 exposure years. For the scenarios, the observations corresponding with this fleet were switched to different companies (namely company 2, 6, 7 and 10). The Table 10 results are for the hierarchical negative binomial model. The *a priori* premiums closely correspond, but *a posteriori* premiums reflect the company differences that are apparent in Figure 3.

Table 8: *Effects of different models on premiums for selected vehicles. Results for hierarchical Poisson, NB and ZIP with fixed p regression models.*

Vehicle Number	*A Priori* (Exp.)	*A Posteriori*	BMF	Acc. Cl. Fleet (Exp.)	Acc. Cl. Veh. (Exp.)
Hierarchical Poisson with random effects for vehicle, fleet and company					
6,645	0.08435 (0.5038)	0.1725	2.05	6 (18.5)	1 (1)
7,006	0.08435 (0.5038)	0.1316	1.56		0 (1)
6,500	0.08435 (0.5038)	0.1329	1.58		0 (1)
Hierarchical NB with random effects for fleet and company					
6,645	0.08383 (0.5038)	0.1435	1.71	6 (18.5)	1 (1)
7,006	0.08383 (0.5038)	0.1435			0 (1)
6,500	0.08383 (0.5038)	0.1435			0 (1)
Hierarchical ZIP with random effects for fleet and company, fixed p					
6,645	0.08241 (0.5038)	0.1484	1.8	6 (18.5)	1 (1)
7,006	0.08241 (0.5038)	0.1484			0 (1)
6,500	0.08241 (0.5038)	0.1484			0 (1)

Note: 'Acc. Cl. Fleet' and 'Acc. Cl. Veh.' are accumulated number of claims at fleet and vehicle levels, respectively. 'Exp.' is exposure at year level, in parenthesis.

Table 9: *Effects of different models on premiums for selected vehicles. Results for ZIP with fleet–specific p and hurdle Poisson model.*

Vehicle	*A Priori* (Exp.)	*A Posteriori*	BMF	Acc. Cl. (Exp.)	Claim Free Years
Hierarchical ZIP with random effects for fleet and company, fleet–specific p					
6,645	0.09051 (0.5038)	0.1306	1.37	6 (18.5)	17
7,006	0.09051 (0.5038)	0.1306			
6,500	0.09051 (0.5038)	0.1306			
Hierarchical hurdle Poisson with random effects for fleet and company					
6,645	0.1098 (0.5038)	0.11	1	6 (18.5)	17
7,006	0.1098 (0.5038)	0.11			
6,500	0.1098 (0.5038)	0.11			

Note: 'Acc. Cl. Fleet' and 'Acc. Cl. Veh.' are accumulated number of claims at fleet and vehicle levels, respectively. 'Exp.' is exposure at year level, in parenthesis.

Table 10: *The effects of switching companies for a selected fleet. Results for negative binomial model.*

	Company				
	2	3	6	7	10
A Priori	0.08695	0.08762	0.08621	0.08831	0.08648
A Posteriori	0.1153	0.08352	0.1071	0.1121	0.1043
BMF	1.33	0.95	1.24	1.27	1.21

5.2.2 Graphical illustrations

Because bonus malus factors are important summary measures of the amount that premiums will increase from one year to the next, we summarize graphically their overall effects. In Figure 4 we consider one model specification, i.e. the negative binomial model from (8). This plot displays per company the BMFs of all its fleets against the average number of claims for the fleet. The latter is calculated as 'total number of claims for the fleet' divided by 'total exposure period for the fleet'. When producing Figure 4, we omitted one fleet with an estimated BMF that exceeded 12 and one fleet with an average claim that was greater than 6.

Figure 4 illustrates company effects. For instance, only few BMFs in company 3 are above 1, whereas company 2 has a majority of fleets with BMF above 1. Plots like the one in Figure 4 are useful tools for pricing actuaries. They are fast and easy instruments to identify the overall riskiness of a collection of fleets as well as the performance of one particular fleet.

Figure 4 can also be used to interpret the size of the BMF. For instance, the omitted fleet with a BMF > 12 was from company 6. Going back to the original data set, we see that this fleet has a capacity of 169 vehicles and reported 123 claims over a period of 153 exposure years. Plotting an outcome, such as the predicted BMF, versus an "explanatory" variable provides a powerful device for interpreting results from a complex model.

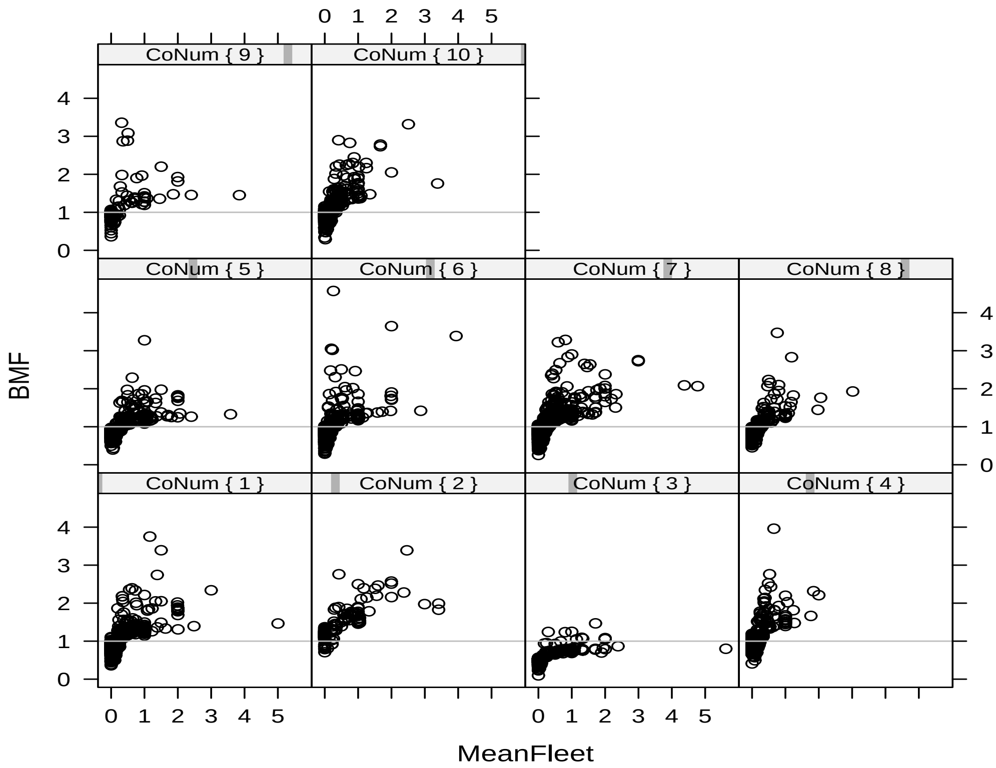
6 Conclusion

This paper presents an analysis of an intercompany data set on claim counts for fleet policies. The data come from 10 Singaporean general insurers who are members of the General Insurance Association in Singapore. Although company effects are widely acknowledged among industry professionals who use such data, our study is the first to formally establish the importance of company effects in the context of a probabilistic model structure.

The framework that we use is a four–level (non–linear) multilevel model of count data. We do not advocate one count distribution at the expense of others but rather use several models that are currently prominent in the literature, including the Poisson, negative binomial, hurdle Poisson and zero-inflated Poisson. We calibrate these models using modern Bayesian estimation techniques.

We find that, in all models considered, there is the importance of accounting for the effects of the various levels. This is true even after the effects of standard rating (explanatory) variables at the different levels in the data set are taken into account. The results also indicate possible

Figure 4: *Bonus–malus factor versus mean claim count, by company. Here, the mean claim count is at the fleet level. This figure shows positive association, on a company by company level.*



different styles for penalizing or rewarding past claims.

To further demonstrate the usefulness of the models, we illustrate how *a priori* rating (using only *a priori* available information) and *a posteriori* updates (taking the claims history into account) for intercompany data can be calculated on a sound statistical basis. A comparison of these calculated premiums results in bonus-malus factors which are important in establishing experience-rated premiums. Insurers, reinsurers and regulators can use the methodology recommended in this paper to study the differences in riskiness among fleets and companies. *A posteriori* predictions for a specific fleet, vehicle or company can be readily calculated from the estimated multilevel models. In future research we hope to address the modeling of hierarchical data on claim sizes instead of frequencies.

References

J.-F. Angers, D. Derjardins, G. Dionne, and F. Guertin. Vehicle and fleet random effects in a model of insurance rating for fleets of vehicles. *ASTIN Bulletin*, 36(1):25–77, 2006.

K. Antonio and J. Beirlant. Actuarial statistics with generalized linear mixed models. *Insurance: Mathematics and Economics*, 40(1):58–76, 2007.

C. Bolancé, M. Guillén, and J. Pinquet. Time-varying credibility for frequency risk models: estimation and tests for autoregressive specifications on random effects. *Insurance: Mathematics and Economics*, 33:273–282, 2003.

J.-P. Boucher, M. Denuit, and M. Guillén. Risk classification for claim counts: mixed Poisson, zero-inflated mixed Poisson and hurdle models. *North American Actuarial Journal*, 11(4): 110–131, 2007.

J.-P. Boucher, M. Denuit, and M. Guillén. *Modeling of insurance claim counts with hurdle distribution for panel data*. Advances in Mathematical and Statistical Modeling: Statistics for Industry and Technology, Birkhäuser Boston, Inc., 2008.

J.-P. Boucher, M. Denuit, and M. Guillén. Number of accidents or number of claims? An approach with zero–inflated Poisson models for panel data. *The Journal of Risk and Insurance*, forthcoming, 2009.

A.C. Cameron and P.K. Trivedi. *Regression analysis of count data*. Cambridge University Press, 1998.

P. Deb, M.K. Munkin, and P.K. Trivedi. Private insurance, selection and health care use: a Bayesian analysis of a Roy–type model. *Journal of Business & Economic Statistics*, 24(4):403–415, 2006.

M. Denuit, X. Maréchal, S. Pitrebois, and J.-F. Walhin. *Actuarial Modelling Of Claim Counts: Risk Classification, Credibility and Bonus-Malus Scales*. Wiley, 2007.

D. Desjardins, G. Dionne, and J. Pinquet. Experience rating scheme for fleets of vehicles. *ASTIN Bulletin*, 31(1):81–105, 2001.

G. Dionne and C. Vanasse. A generalization of actuarial automobile insurance rating models: the Negative Binomial distribution with a regression component. *ASTIN Bulletin*, 19:199–212, 1989.

E.W. Frees and E.A. Valdez. Hierarchical insurance claims modeling. *Journal of the American Statistical Association*, 103(484):1457–1469, 2008.

E.W. Frees, V.R. Young, and Y. Luo. A longitudinal data analysis interpretation of credibility models. *Insurance: Mathematics and Economics*, 24(3):229–247, 1999.

A. Gelman and J. Hill. *Applied Regression and Multilevel (Hierarchical) Models*. Cambridge University Press, Cambridge, 2007.

H. Goldstein. *Multilevel Statistical Models*. Oxford University Press, 2003.

J.C. Hickman and L. Heacox. Credibility theory: the cornerstone of actuarial science. *North American Actuarial Journal*, 3(2):1–8, 1999.

B. Iverson, J. Luff, S. Siegel, and R. Stryker. The SOA: A place for research – answer to all your questions about the research done by the SOA. *The Actuary Magazine*, October/November, 2007.

W.S. Jewell. The use of collateral data in credibility theory: a hirerachical model. *Giornale dell'Instituto Italiano degli Attuari*, 38:1–6, 1975.

R. Kaas, M. Goovaerts, J. Dhaene, and M. Denuit. *Modern actuarial risk theory using R*. Springer–Verlag, Berlin, 2008.

I.G.G. Kreft and J. deLeeuw. *Introducing Multilevel Modeling*. Sage Publications, London, 1998.

A. Lee, K. Wang, J. Scott, K. Yau, and G. McLachlan. Multi-level zero-inflated Poisson regression modelling of correlated count data with excess zeros. *Statistical Methods In Medical Research*, 15(1):47–61, 2006.

J. Lemaire. *Bonus–malus systems in automobile insurance*. Springer–Verlag, New York, 1995.

R. Norberg. Hierarchical credibility: analysis of a random effect linear model with nested classification. *Scandinavian Actuarial Journal*, pages 204–222, 1986.

J. Pinquet. Allowance for cost of claims in bonus-malus systems. *ASTIN Bulletin*, 27(1):33–57, 1997.

J. Pinquet. Designing optimal bonus-malus systems from different types of claims. *ASTIN Bulletin*, 28(2):205–229, 1998.

J. Pinquet, M. Guillén, and C. Bolancé. Allowance for age of claims in bonus-malus systems. *ASTIN Bulletin*, 31(2):337–348, 2001.

S.W. Raudenbush and A.S. Bryk. *Hierarchical Linear Models: Applications and Data Analysis Methods*. Sage Publications, Thousand Oaks, 2002.

T.A.B. Snijders and R.J. Bosker. *Multilevel Analysis: an introduction to basic and advanced multilevel modeling*. Sage Publications, London, 1999.

D.J. Spiegelhalter, N.G. Best, B.P. Carlin, and A. Van der Linde. Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society, Series B*, 64(3):583–639, 2002.

B. Sundt. A multi–level hierarchical credibility regression model. *Scandinavian Actuarial Journal*, 25–32, 1980.

G. Taylor. Credibility analysis of a general hierarchical model. *Scandinavian Actuarial Journal*, pages 1–12, 1979.

R. Winkelmann. *Econometric Analysis of Count Data*. Springer-Verlag, Berlin, 2003.

K. Yau, K. Wang, and A. Lee. Zero-inflated negative binomial mixed regression model. *Biometrical Journal*, 45(4):437–452, 2003.

K. Yip and K. Yau. On modeling claim frequency data in general insurance with extra zeros. *Insurance: Mathematics and Economics*, 36:153–163, 2005.

Appendix. Categorization Tables

Table 11: *Vehicle level categorizations*

Covariates	Categorization	
Vehicle Age	0<=VAgeEntry<=4	Reference
	VAgeEntry>4	
Cubic Capacity	0<VehCapCubic<=1500	Reference
	VehCapCubic>1500	
Tonnage	0<Tonnage<=2	Reference
	2<Tonnage<=8	
	8<Tonnage	
TLengthEntry	TLengthEntry=0	Reference
	TLengthEntry>0	
TLInFleet	TLInFleet =0	Reference
	TLInFleet > 0	

Table 12: *Fleet level categorizations*

Covariates	Categorizations	
AvNCD	AvNCD=0	
	AvNCD>0	Reference
AvTLengthEntry	AvTLengthEntry=0	Reference
	0<AvTLengthEntry	
AvTLength	AvTLength<=0.7	Reference
	AvTLength> 0.7	
AvVAge	0<= AvVAgeEntry <=4	
	4<AvAgeEntry	
FleetChange	Not defined	Reference
	0 < FlChange < 1	
	FlChange = 1	
	FlChange > 1	

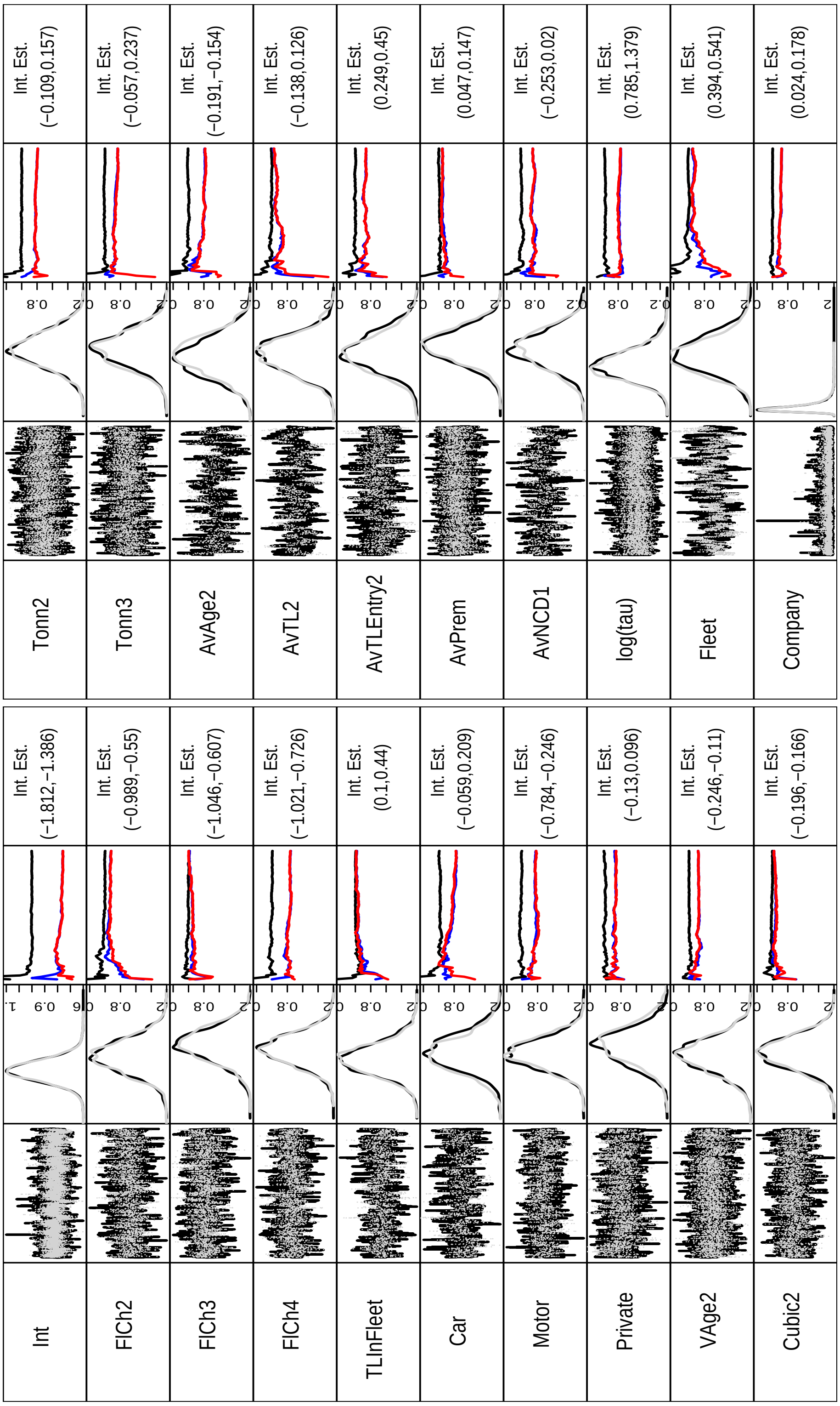


Figure 5: *Structure (8): convergence diagnostics for the simulated Bayesian estimates. For parameter estimates associated with each variable, the figure shows a trace–plot, density, BGR convergence diagnostics and 95% credibility interval.*